Program Analysis with Dynamic Change of Precision

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Joint work with
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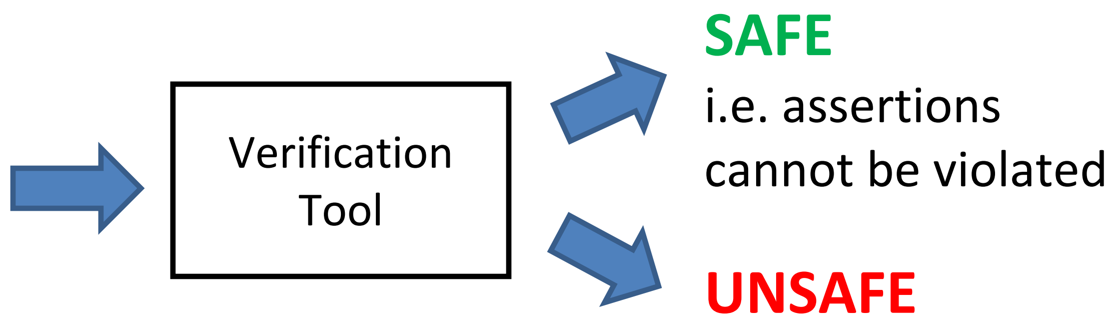
Automatic Software Verification

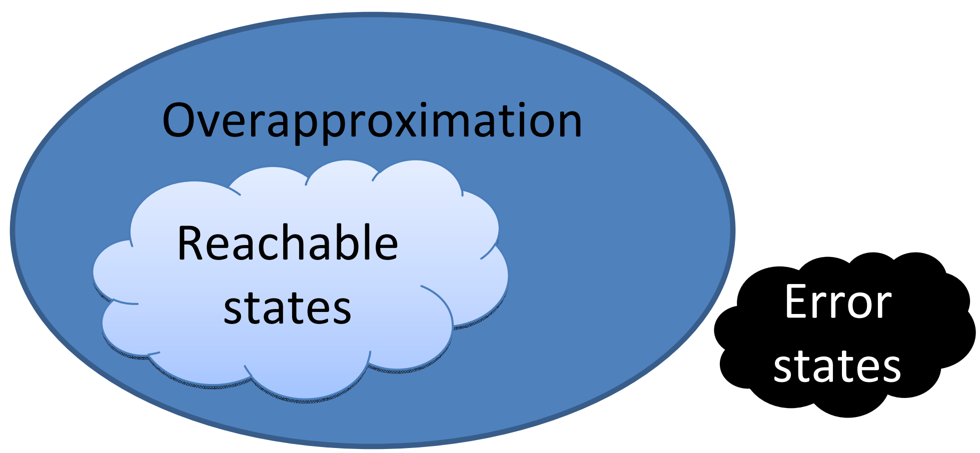
C program

```c
int main() {
  int a = foo();
  int b = bar(a);

  assert(a == b);
}
```



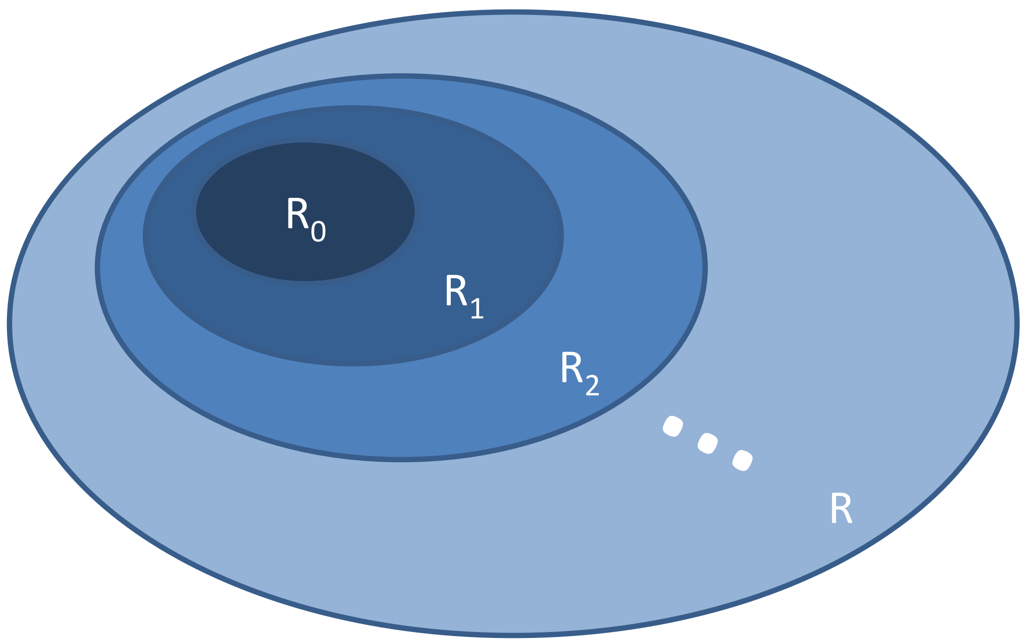
General method:
Create an overapproximation of the program states



Software Verification by Model Checking

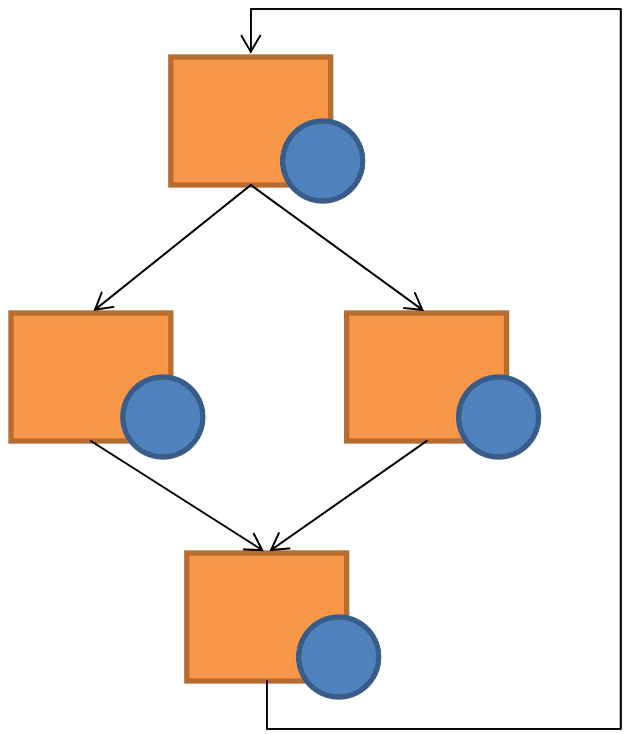
[Clarke/Emerson, Sifakis 1981]

Iterative fixpoint (forward) post computation

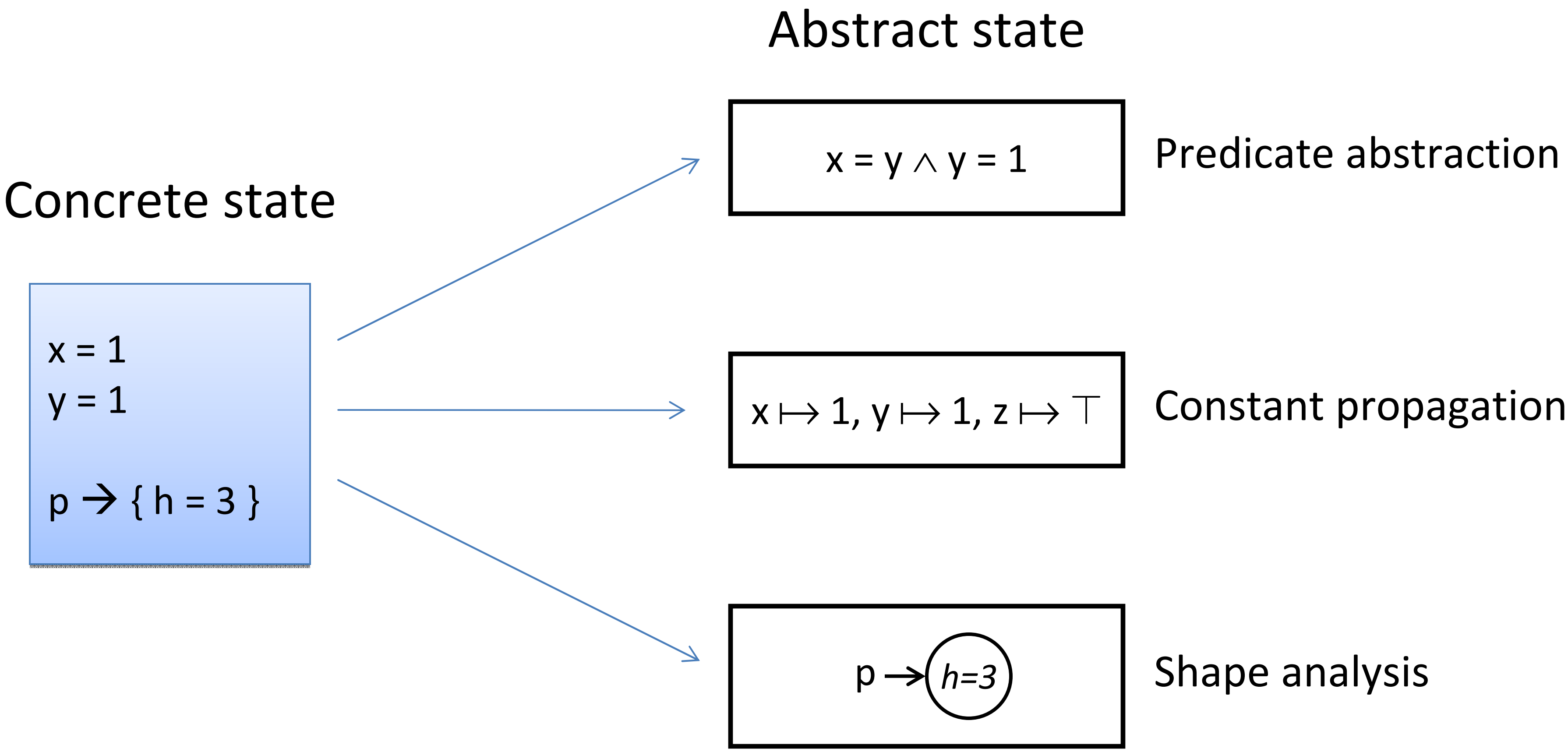


Software Verification by Data-flow Analysis

Fixpoint computation on the CFG



Abstraction



Few explicit values
→ Explicit domain

Many values
→ Predicate abstraction

```
int f = 0, x = 1, y = 0;
while (x > 0) {
  if (f == 0) { x = 50; f = 1; }
  else        { x--; y++; }
}
assert(y == 50);
```

Fully Explicit Analysis

```
int f = 0, x = 1, y = 0;
while (x > 0) {
   if (f == 0) { x = 50; f = 1; }
   else        { x--; y++; }
}
assert(y == 50);
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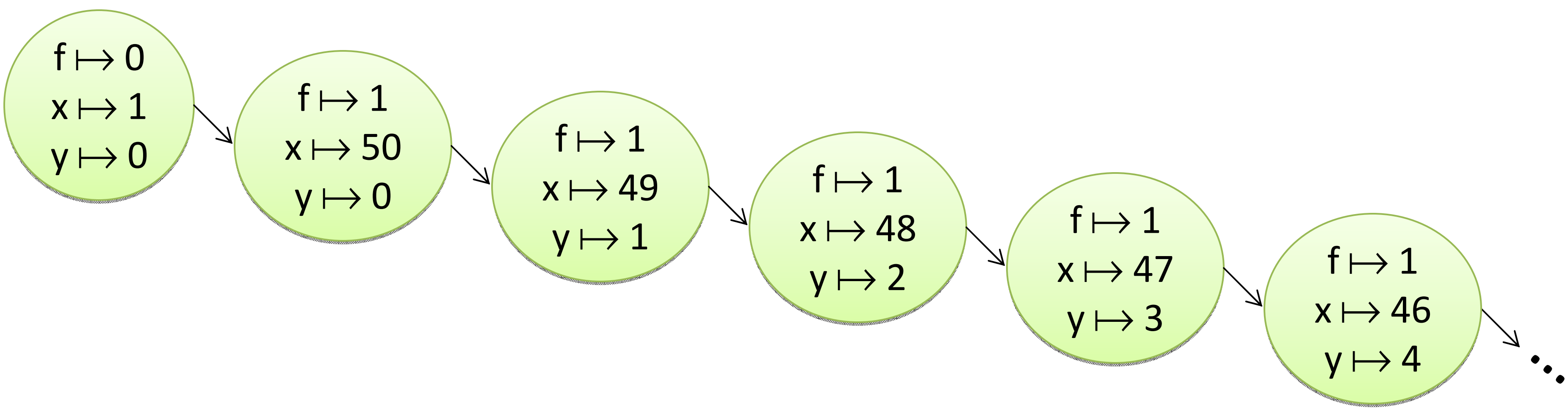

\+ cheap to compute

\- many states

```
int f = 0, x = 1, y = 0;
while (x > 0) {
  if (f == 0) { x = 50; f = 1; }
  else        { x--; y++; }
}
assert(y == 50);
```

Combined Analysis

- Start with explicit
- Precision of explicit: threshold on number of diff. values

  E.g. π(x) = 3

  - Precision of predicate: set of tracked predicates

    E.g. $\pi = \{ x + y = 50,\ x \geq 0, x = 0\}$

  - Switch to predicates when the explicit threshold is hit

```
int f = 0, x = 1, y = 0;
while (x > 0) {
   if (f == 0) { x = 50; f = 1; }
   else        { x--; y++; }
}
assert(y == 50);
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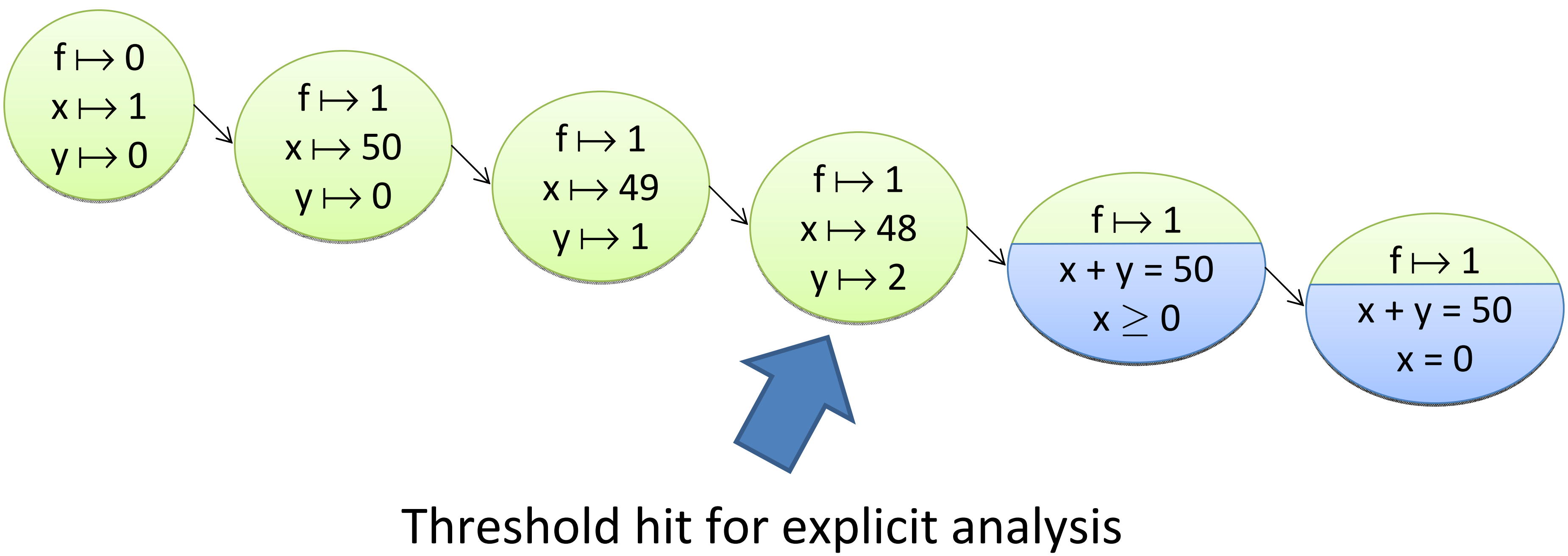
Combined Analysis



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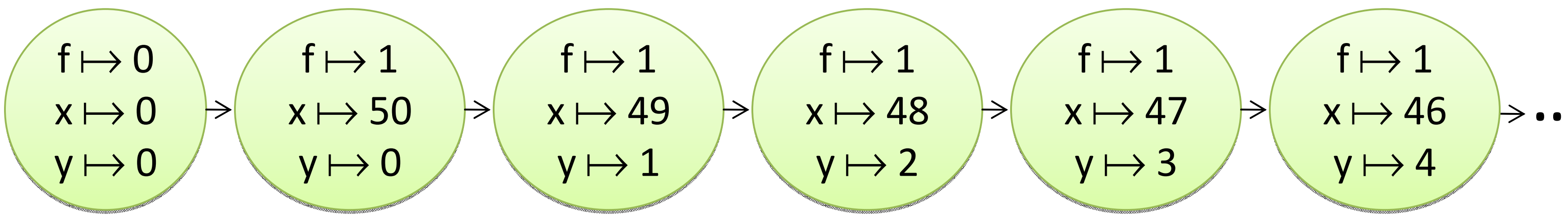
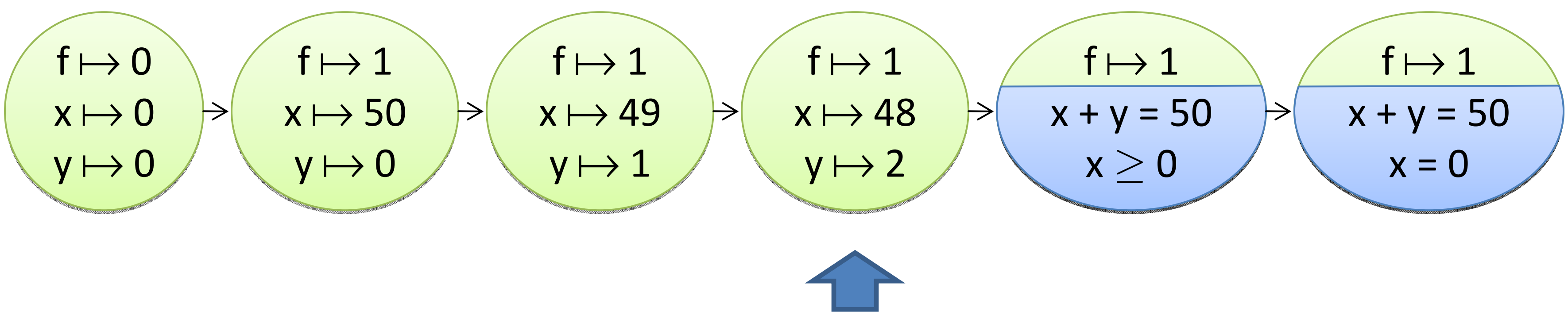
Fully Explicit Analysis



Combined Analysis



Motivation

- Flexible combination of abstract domains
- Dynamically update their respective *precisions*
  - precision: set of predicates, variables, etc. to track
  - e.g. switch on/off analyses
  - e.g. use different analyses for different variables
  - …

Software Model Checking

Reached, *Frontier* := { e_0 }

while *Frontier* ≠ ∅ do

  remove *e* from *Frontier*

  for each *e'* ∈ **post**(*e*) do

    if ¬ **stop**(*e'*, *Reached*) add *e'* to *Reached*, *Frontier*

return *Reached*

Configurable Program Analysis
[Beyer/Henzinger/T 2007]

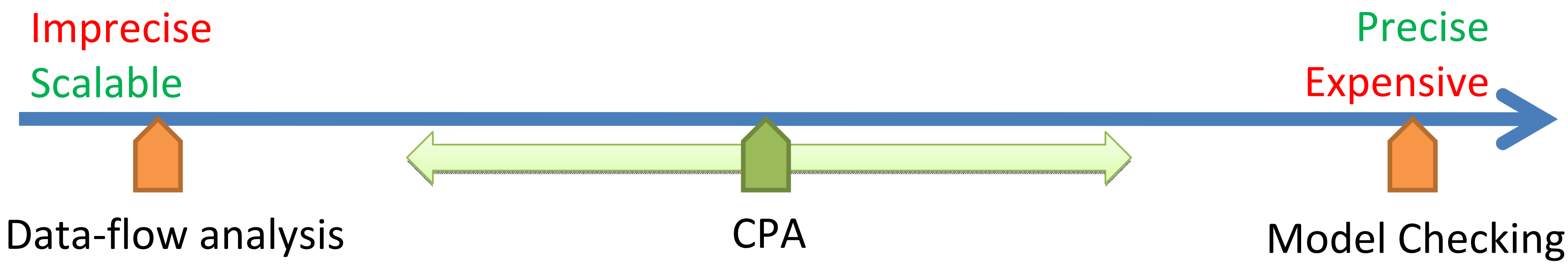
Reached, *Frontier* := { e_0 }
while *Frontier* $\neq \emptyset$ do
  remove *e* from *Frontier*

  for each $e' \in$ **post**(*e*) do
    for each $e'' \in$ *Reached* do
      e''_{new} := **merge**(*e'*, *e''*)
      if $e''_{new} \neq e''$ then
        replace *e''* in *Reached*, *Frontier* by e''_{new}
    if $\neg$ **stop**(*e'*, *Reached*) add *e'* to *Reached*, *Frontier*

return *Reached*

Configurable Program Analysis

- Better combination of abstractions
  ➔ Configurable Program Analysis [CAV07]



Unified framework that enables intermediate algorithms

Configurable Program Analysis
[CAV 2007]

Reached, *Frontier* := { e_0 }
while *Frontier* $\neq \emptyset$ do
  remove *e* from *Frontier*

  for each $e' \in$ **post**(*e*) do
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      if $e''_{new} \neq e''$ then
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    if $\neg$ **stop**(*e'*, *Reached*) add *e'* to *Reached*, *Frontier*

return *Reached*

Configurable Program Analysis (CPA+)
with Dynamic Precision Adjustment

[ASE 2008]

Reached, *Frontier* := { (e_0 , π_0) }

while *Frontier* $\neq \emptyset$ do

  remove (e , π) from *Frontier*

  ($\hat{e}$, π_{new}) = **prec**(e, π, *Reached*)

  for each $e' \in$ **post**($\hat{e}$, π_{new}) do

    for each (e'', π'') $\in$ *Reached* do

      e''_{new} := **merge**(e', e'', π_{new})

      if $e''_{new} \neq e''$ then

        replace (e'' , π'') in *Reached*, *Frontier* by (e''_{new} , π_{new})

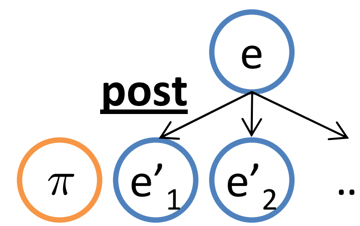
    if $\neg$ **stop**(e', *Reached*, π_{new}) add (e', π_{new}) to *Reached*, *Frontier*

return *Reached*

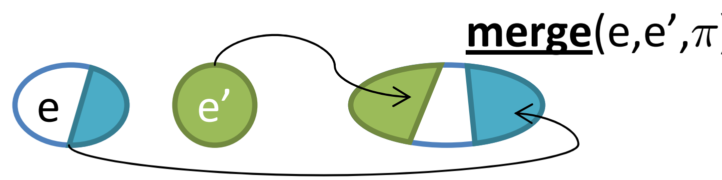
CPA+

Configurable program analysis with dynamic precision adjustment:

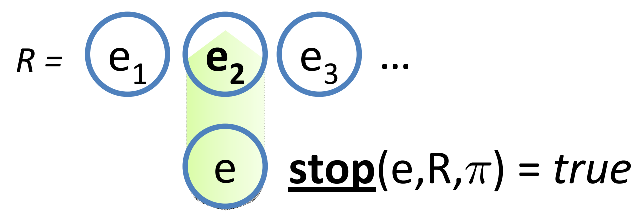
- concrete system $(C, c_0, \rightarrow)$
- abstract domain $(E, \top, \bot, \sqsubseteq, \sqcup)$
- a set of precisions Π
- concretization function $\gamma: E \rightarrow 2^C$
- transfer function **post** $\subseteq E \times \Pi \rightarrow 2^E$



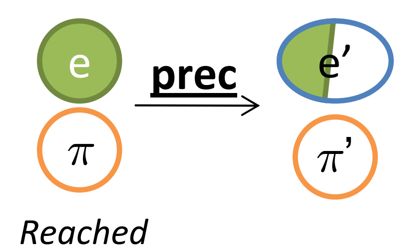
- merge operator **merge**: $E \times E \times \Pi \rightarrow E$



- termination check **stop**: $E \times 2^E \times \Pi \rightarrow \mathbb{B}$



- precision adjustment: **prec**: $E \times \Pi \times 2^{E \times \Pi} \rightarrow E \times \Pi$



Note: Operators are required to be soundly overapproximating

CPA+

Configurable program analysis with dynamic precision adjustment:

- concrete system $(C, c_0, \rightarrow)$
- abstract domain $(E, \top, \bot, \sqsubseteq, \sqcup)$
- a set of precisions Π
- concretization function $\gamma: E \rightarrow 2^C$

- transfer function **post** $\subseteq E \times \Pi \rightarrow 2^E$

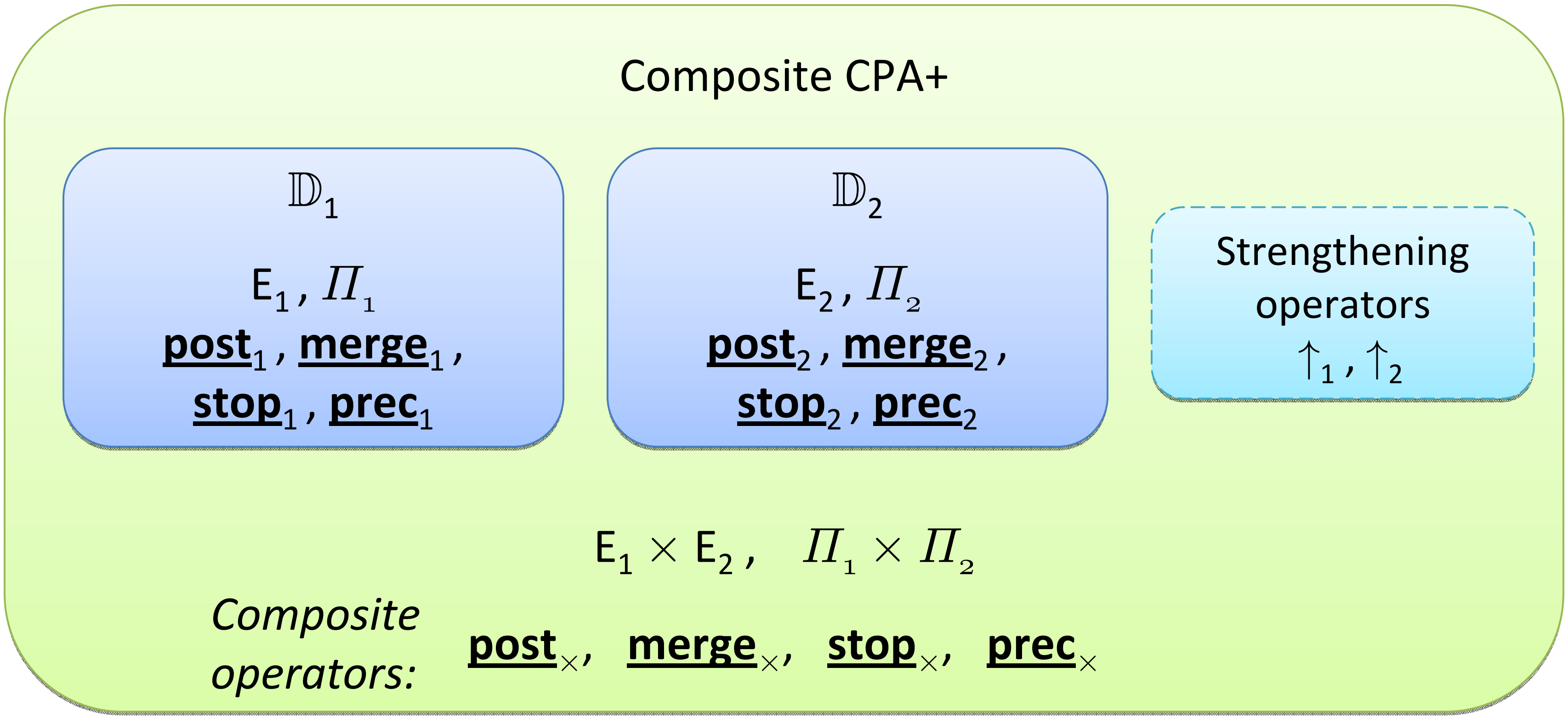
- merge operator **merge**: $E \times E \times \Pi \rightarrow E$
  - $merge^{sep}(e,e',\pi) = e'$
  - $merge^{join}(e,e',\pi) = e \sqcup e'$

- termination check **stop**: $E \times 2^E \times \Pi \rightarrow \mathbb{B}$
  - $stop^{sep}(e,R,\pi) = \exists e' \in R, e \sqsubseteq e'$
  - $stop^{join}(e,R,\pi) = e \sqsubseteq \sqcup R$

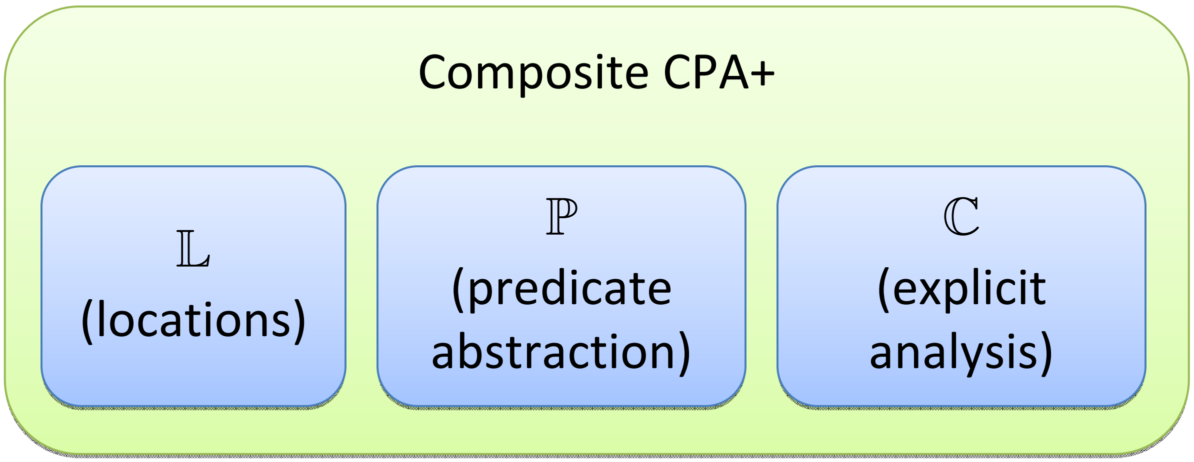
- precision adjustment: **prec**: $E \times \Pi \times 2^{E \times \Pi} \rightarrow E \times \Pi$

Note: Operators are required to be soundly overapproximating

Composite CPA+



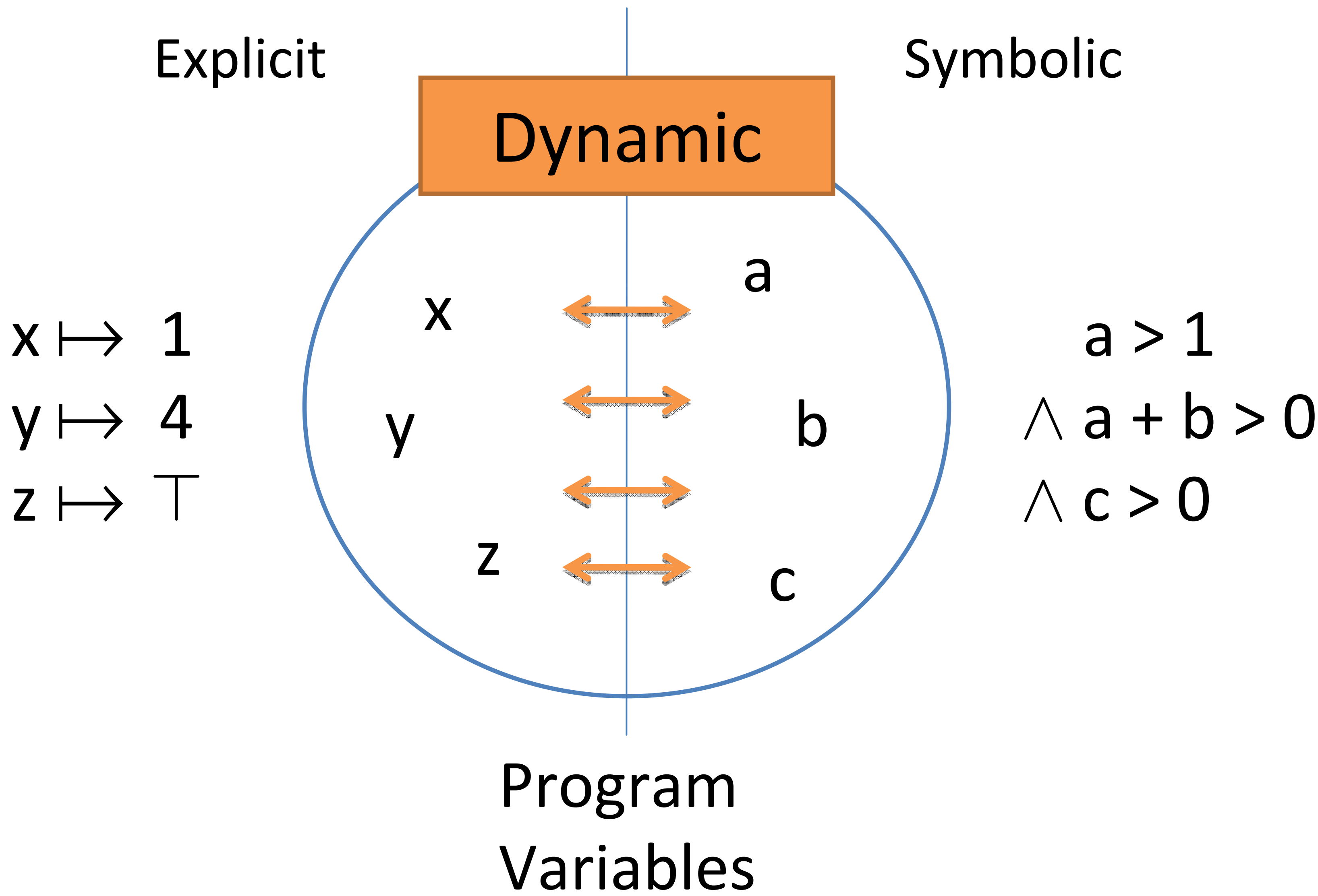
Example: Predicate Abstraction + Explicit



Example of composite abstract element:

$(6 , \quad x > 0 \wedge x = y , \quad \{ i \mapsto 2, x \mapsto \top, y \mapsto \top \})$

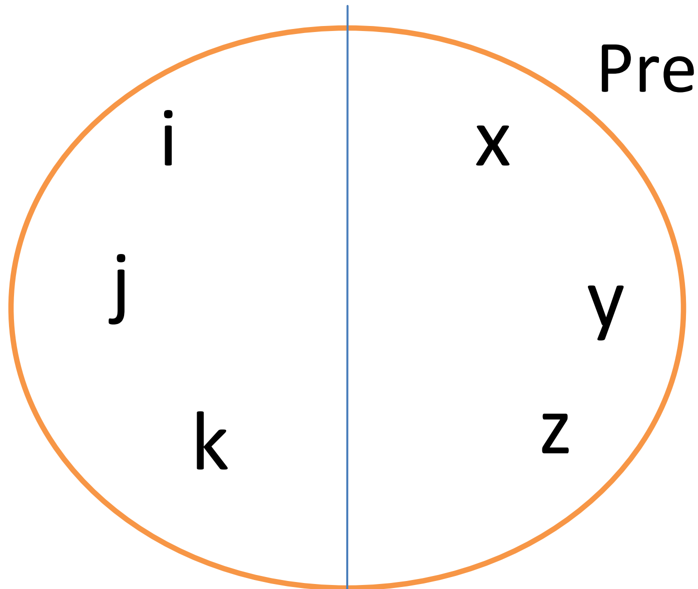
Combined Abstraction



Composite Precision

Explicit

$i \mapsto 1$
$j \mapsto 4$
$k \mapsto \top$



Predicate Abstraction

$x > 1$
$\wedge\ x + y > 0$
$\wedge\ z > 0$

Precision

$\pi = \{ (i,5), (j,3), (k,5), (x,0), (y,0), (z,0) \}$

$\pi = \{ x > 1, z < 7, x + y > 0, z > 0 \}$

Domain and Precisions

CPA+	**Abstract domain**	**Precisions / Precision Adjustment**
$\mathbb{P}$	Predicate Abstraction $E = 2^{\mathcal{P}}$ e.g. $e = \{ x < 3, y > 0 \}$	*Set of tracked predicates* $\Pi = 2^{\mathcal{P}}$ $prec_{\mathbb{P}}(e,\pi,R) = (e,\pi)$
$\mathbb{C}$	Explicit Analysis $E = [X \rightarrow \mathcal{Z}]$ $\mathcal{Z} = \mathbb{Z} \cup \{\perp,\top\}$ e.g. $e = \{ x \mapsto 2, y \mapsto \top, \ldots \}$	*Max. number of diff. values per variable* $\Pi = [X \rightarrow \mathbb{N}]$ $prec_{\mathbb{C}}(e,\pi,R) = (e',\pi)$ if for all $x \in X$: if $\lvert R(x) \rvert \geq \pi(x)$, then $e'(x) = \top$ otherwise, $e'(x) = e(x)$

Example: Predicate Abstraction + Explicit

Idea:

Dynamically choose between explicit & predicate abstraction

- Too many explicit values $\rightarrow$ predicate abstraction

- Use explicit values to infer predicates

Implementable in the composite <u>prec</u> operator

Note: explicit analysis $\approx$ testing on some variables

Few explicit values → Explicit domain

Too many values, or need of symbolic representation → Predicate abstraction

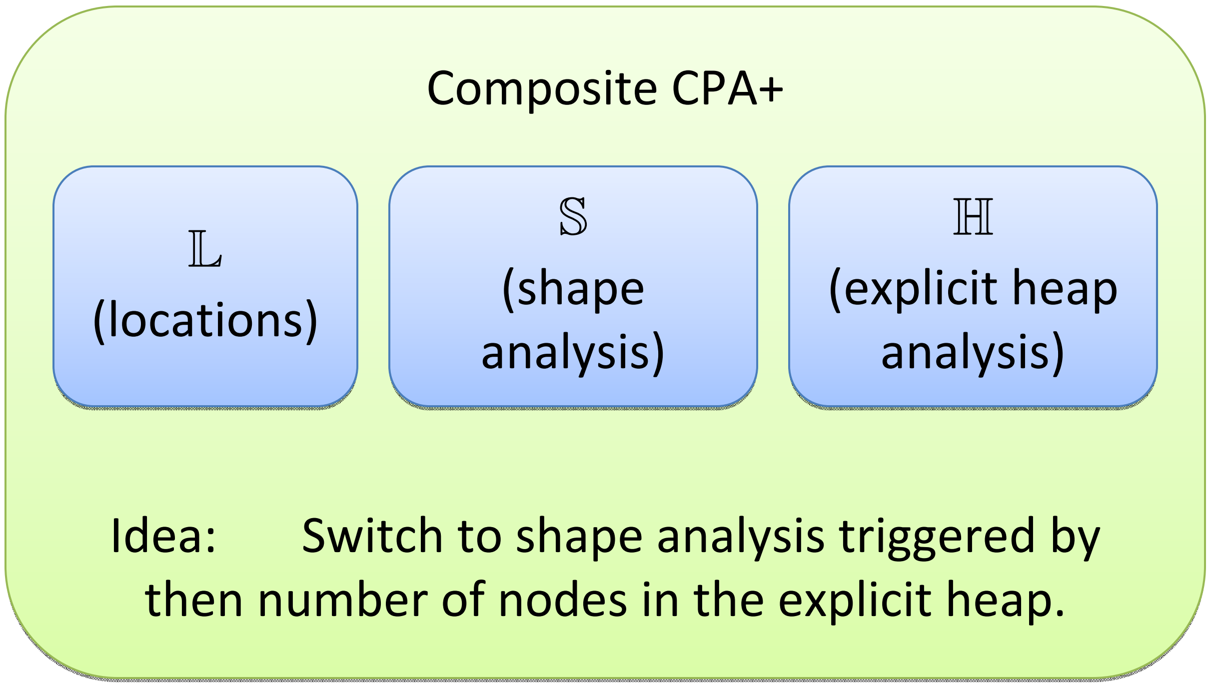
```
int main() {
  int st = 0, ok = 0, cmd = readcmd();
  int *a = getarray(); int n = length(a), p = 0;
  while (1) {
    switch (st) {
      case 0: if (cmd == 177) st = 1; else st = 3; break;
      case 1: st = 3; cmd = readcmd(); break;
      case 3: if (cmd == 78) { ok = 1; goto cont; }
         else { cmd = readcmd(); st = 0; }
      ...
    }
  }
cont:  assert(ok == 1);
  while (p < n) { a[p] = 0; p++; }
  return 0;
}
```

Program Threshold	**Pred. only** k = 0	**Predicate + Explicit** k = 1	k = 5	**Explicit only** k = ∞
ex1	0.46 s	0.17 s	0.21 s	—
ex2	0.43 s	0.16 s	0.21 s	1.00 s
loop1	25.20 s	26.01 s	22.78 s	0.16 s
loop2	279.84 s	277.07 s	258.79 s	0.44 s
square	—	—	—	0.08 s

Program Threshold	**Pred. only** k = 0	**Predicate + Explicit** k = 1	k = 2
s3_clnt (total)	75.11 s	10.40 s	18.42 s
s3_srvr (total)	536.79 s	24.23 s	34.40 s

Precision for explicit analysis: $\pi_{\mathbb{C}}(x) = k$

Another combination: Shapes + Explicit heap



Conclusion

- Unifying Data-Flow Analysis and Model Checking

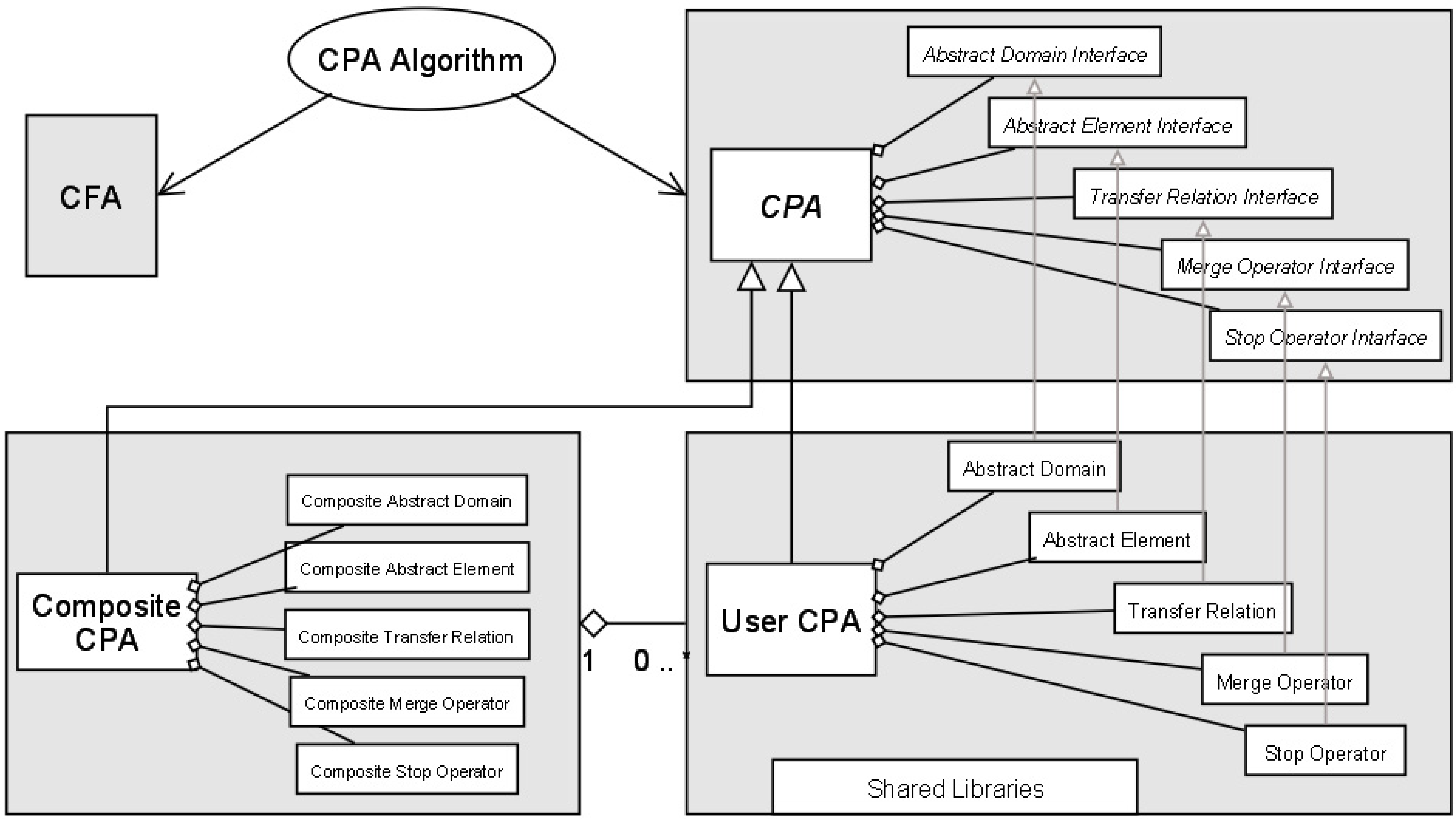
- Framework to express change of precision during the analysis ($\neq$ refinement)
  - → Useful when composing *existing* analyses
  - → Make combination more effective/precise

- References
  - → CAV'07: Configurable Software Verification
  - → ASE'08: PA with Dynamic Precision Adjustment

Current Development: **CPAchecker**

- Tool that implements these concepts

CPAchecker -- Design



CPAchecker -- Architecture and Flow