Master's Thesis

Solver-based Analysis of Memory Safety using Separation Logic

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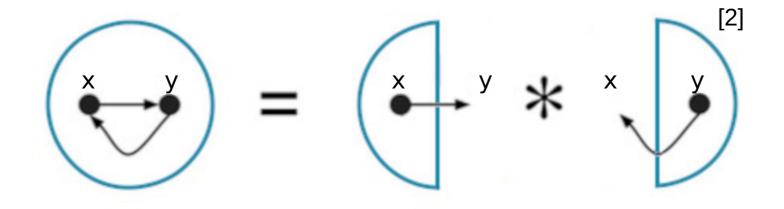


"First, [SL] merges with the scientific-engineering model the programmer uses to understand and build the software. [...]

Secondly, the proof theory developed to check software using SL is based on rules for scaling the reasoning task [...]" (Pym et al. [1])

Background

• Two memory cells "separately in memory"



- Corresponding formula: $x \mapsto y \quad * \quad y \mapsto x$
- Implicit assertion by spatial conjuntion * that $x \neq y$

Background

- Based on Symbolic Heaps, SL extends Hoare Logic [3, 4] by spatiality
- Rule of constancy does <u>not</u> hold for SL

$$\frac{\{x \mapsto 0\} \ast x = 42 \ \{x \mapsto 42\}}{\{x \mapsto 0 \land y \mapsto 0\} \ast x = 42 \ \{x \mapsto 42 \land y \mapsto \emptyset\}} \not [x = y]$$

• Spatial conjunction works out

$$\frac{\{x \mapsto 0\} \ *x = 42 \ \{x \mapsto 42\}}{\{x \mapsto 0 * y \mapsto 0\} \ *x = 42 \ \{x \mapsto 42 * y \mapsto 0\}}$$

Pointer Analysis

- Based on symbolic execution
- CPA implementation of the CPAchecker framework
- Transformation of C code to SL formulae using JavaSMT
- Check formulae for memory safety properties:
 - Invalid read and write (\perp^R , \perp^W)
 - Invalid free (\perp^F)
 - Memory leak (\perp^L)

Pointer Analysis - Abstract Domain

$$\begin{split} \kappa &:= \text{typical constants} & \text{Constants} \\ Var &:= x, y, \dots & \text{Program variables} \\ Var' &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ E, F, G &:= x', y', \dots & \{+, \cdot, <<, >>, \bullet\} & \text{Symbolic variables} \\ F &:= true \mid E = F \mid E &\subset F \mid \neg C & \text{Constraints} \\ \Pi &:= C \mid \Pi \wedge \Pi & \text{Pure formulae} \\ \Sigma &:= emp \mid E \mapsto F \mid \Sigma * \Sigma & \text{Spatial formulae} \\ \mathcal{H} &:= \Pi \wedge \Sigma & (\text{Quantifier-free}) & \text{Symbolic heaps} \end{split}$$

Grammar of SL formulae

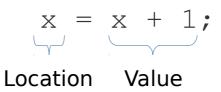
Pointer Analysis - Language

$$\begin{split} \tau &:= \text{typical types excluding floats} & \text{Types} \\ \kappa &:= \text{typical constants} & \text{Constants} \\ \epsilon_{\ell} &:= x, y, \dots \mid * \epsilon \mid a[\epsilon] \mid a.b & \text{Left-hand sides} \\ \epsilon &:= \kappa \mid \epsilon_{\ell} \mid \& \epsilon_{\ell} \mid \epsilon \odot \epsilon \mid f(\epsilon_{r}, \dots, \epsilon_{r}) \mid (\tau) \epsilon & \text{Right-hand sides} \\ s &:= \tau \mid a \mid \epsilon \mid \epsilon_{\ell} = \epsilon \mid \{s\} \mid s; s & \text{Statements} \\ \mid while(\epsilon) \mid s \mid if(\epsilon) \mid s \mid else \mid s \end{split}$$

Basic programming language inspired by C

Pointer Analysis - Locations and Values

• C assignment statement of the form $\epsilon_{\ell} = \epsilon$



• Transformation of both to the representative formula

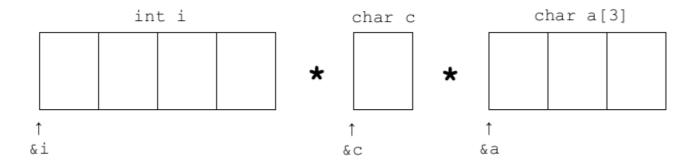
$$\Sigma \models \epsilon_{\ell} \Downarrow_{l} E \qquad \Sigma \models \epsilon \Downarrow_{v} E$$

Pointer Analysis - Memory Model

• Syntactic sugar for memory segments

 $E \mapsto F_0, \dots, F_n := (E \mapsto F_0) * \dots * (E + n \mapsto F_n)$

• Memory as collection of byte-sequences



Corresponding formula (simplified)

 $\&i\mapsto \underbrace{i^0,i^1,i^2,i^3}_{\text{bytes (little endian)}} * \ \&c\mapsto c \ * \ \&a\mapsto a[0],a[1],a[2]$

Pointer Analysis - Memory Access

Allocation check

$$\begin{split} \Sigma \models Allocated(E) \\ & \longleftrightarrow \\ \Sigma \models \exists \Sigma', F.(E = F \land \Sigma = \Sigma' * F \mapsto -) \end{split}$$

- Frame and antiframe
- Frame inference and antiframe abduction together refered to as *Bi-Abduction* [5]
- Bi-Abduction not represented in SL-COMP 2019 [6]

Pointer Analysis - Memory Access

Allocation check using a SL solver (without Bi-Abduction)

$$\neg SAT(\Sigma * E \mapsto -)$$

• Dereferencing

 $\begin{array}{ll} Deref(emp,\ E) := nil \\ Deref(\Sigma,nil) := nil \\ \hline \neg SAT(F\mapsto G*E\mapsto -) \\ Deref(F\mapsto G*\Sigma,\ E) := \text{if } F\mapsto G\models Allocated(E) \\ & \text{then } G \text{ else } Deref(\Sigma,E) \end{array}$

- As part of the CPA formalism, CFA provides different kinds of edges:
 - Statement edge
 - Assumption edge
 - Function call edge
 - ...
- For each of them, rules are defined to track the memory manipulation

• Statement edge: dynamic allocation and assignment

$$\frac{\Sigma * E \mapsto -^{0}, \dots, -^{n-1} \models \epsilon_{\ell} \Downarrow_{l} E \qquad n = sizeof(\epsilon_{\ell})}{\Sigma * E \mapsto -^{0}, \dots, -^{n-1} \models \epsilon \Downarrow_{v} \kappa \qquad \Sigma \models malloc(\epsilon) \Downarrow_{v} x}$$

$$\overline{\{\Pi \land \Sigma * E \mapsto -^{0}, \dots, -^{n-1}\}} \epsilon_{\ell} = malloc(\epsilon) \{\frac{\Pi \land \Sigma * E \mapsto x^{0}, \dots, x^{n-1}}{* x \mapsto -^{0}, \dots, -^{\kappa-1}}\}}$$

$$\frac{F \text{ pointing to n bytes}}{\{\Pi \land \Sigma\}} e_{\ell} = \epsilon \{\bot^{R}\}} \text{ Assign}^{\text{InvR}} \qquad \frac{\Sigma \models \epsilon_{\ell} \Downarrow_{l} nil}{\{\Pi \land \Sigma\}} \epsilon_{\ell} = \epsilon \{\bot^{R}\}} \text{ Assign}^{\text{InvW}}$$
Similar rule for statements other than assignments

• Statement edge: dynamic deallocation

$$\begin{split} \frac{\Sigma * E \mapsto -^{0}, \dots, -^{n-1} \models \epsilon \Downarrow_{v} E \quad n = segmentSize(E)}{\{\Pi \land \Sigma * E \mapsto -^{0}, \dots, -^{n-1}\} \operatorname{free}(\epsilon) \{\Pi \land \Sigma\}} \text{ Free} \\ \frac{\Sigma \models \epsilon \Downarrow_{v} E \quad \Sigma^{h} \nvDash Allocated(E)}{\{\Pi \land \Sigma^{s} * \Sigma^{h}\} \operatorname{free}(\epsilon) \{\bot^{F}\}} \text{ Free}^{\operatorname{Inv1}} \\ \frac{\Sigma \models \epsilon \Downarrow_{v} E \quad segmentSize(E) = -1}{\{\Pi \land \Sigma\} \operatorname{free}(\epsilon) \{\bot^{F}\}} \text{ Free}^{\operatorname{Inv2}} \end{split}$$

- segmentSize(E) determines the size of an allocated segment for a given start address, -1 otherwise
- Symbolic heap can be subdivided into heap Σ^h and stack Σ^s part as referred to in the context of C

• Assumption edge: feasability check

$$\frac{\Sigma \models \epsilon \Downarrow_v C \quad \Pi \land C \text{ is SAT}}{\{\Pi \land \Sigma\} \epsilon \{\Pi \land C \land \Sigma\}} \text{ Assume}^+$$

- Ensures termination of analysis
- SMT solver sufficient for satisfiability check

Pointer Analysis - Memory Leak

Dropped values might lead to leakage (assignment, scope, free())

1 char *p = 0;
2 {
3 char *q = malloc(1);
4 p = q;
5 }

$$\sum^{s} \sum^{h}$$

malloc! malloc! $\rightarrow 0$

Pointer Analysis - Memory Leak

 Dropped values pointing to an allocated heap cell have to be checked for reachability

E is active heap address

 $Leak(\Sigma^{s} * \Sigma^{h}, E) := (\Sigma^{h} \models Allocated(E)) \land \neg reachable(\Sigma^{s} * \Sigma^{h}, E)$

direct access via stack

 $reachable(\Sigma^s * \Sigma^h, E) := (\Sigma^s \models Allocated(E))$

 $\vee (\exists F, G. (F \mapsto G \land E = G \land reachable(\Sigma^s * \Sigma^h, F)))$

alias F exists

check alias recursively

 Remember already visited aliases to handle cycles (here: left out for readability)

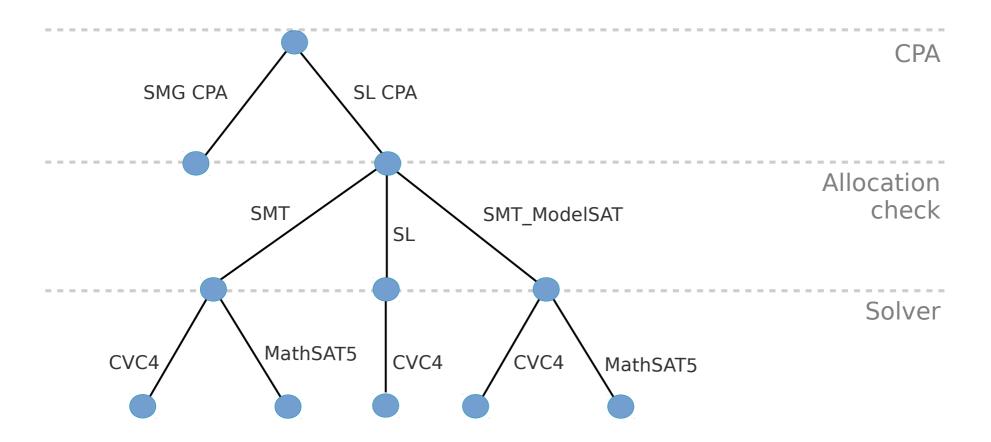
- Allocation checks for each heaplet might lead to overhead
- "Simulation" through SMT

$$\neg SAT(F \mapsto G * E \mapsto -) \Longleftrightarrow SAT(F = E)$$

Can be further optimized to a single solver call using model generation

Evaluation

• Comparison of the approach (SL CPA) to SMG CPA (cf. [7])



Evaluation - Test-sets

- SV-Benchmarks
 - memsafety-ext3 (18/18)¹
 - memsafety-ext2 (2/10)²
- CPAlien test-set (16/21)³
- 36 problems solved
- including function calls, singly-linked lists and structures

1: https://github.com/sosy-lab/sv-benchmarks/tree/master/c/memsafety-ext3

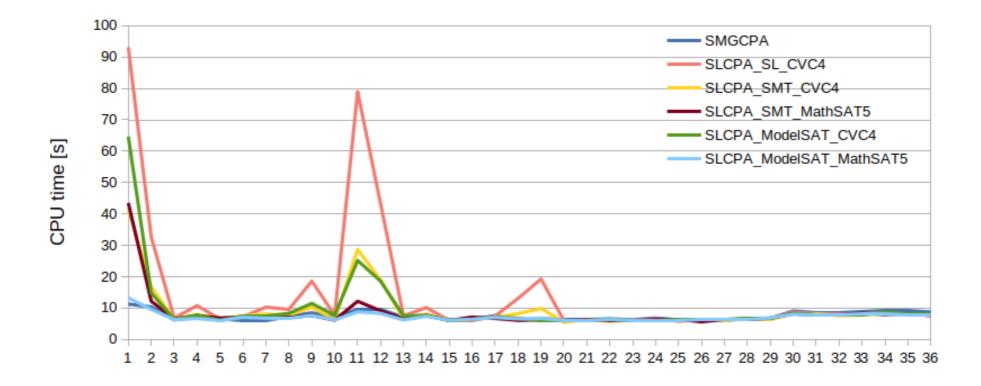
^{2:} https://github.com/sosy-lab/sv-benchmarks/tree/master/c/memsafety-ext2

^{3:} https://github.com/sosy-lab/cpachecker/tree/trunk/test/programs/cpalien

Evaluation - Execution Environment

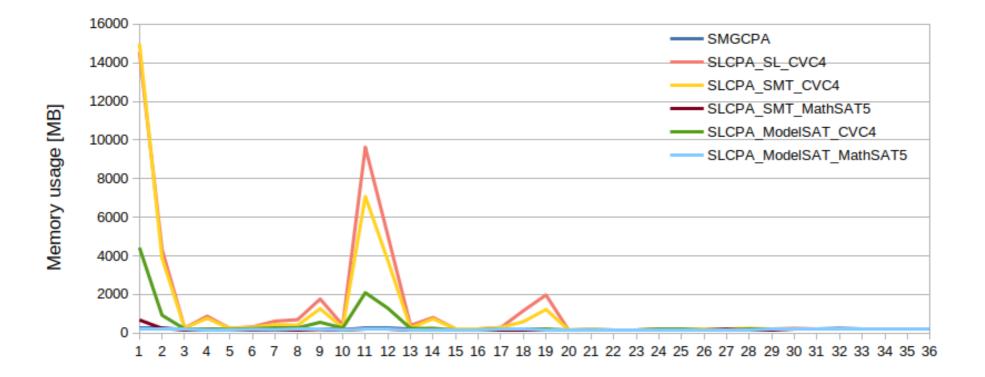
- Intel Xeon E3-1230 v5 CPUs, 3.40 GHz CPU frequency, 33 GB RAM
- Run on two CPU cores, limited to 90s execution time and 15GB RAM
- Measurments using BENCHEXEC
- **Branch:** sl-integration0:r34981

Evaluation - CPU time



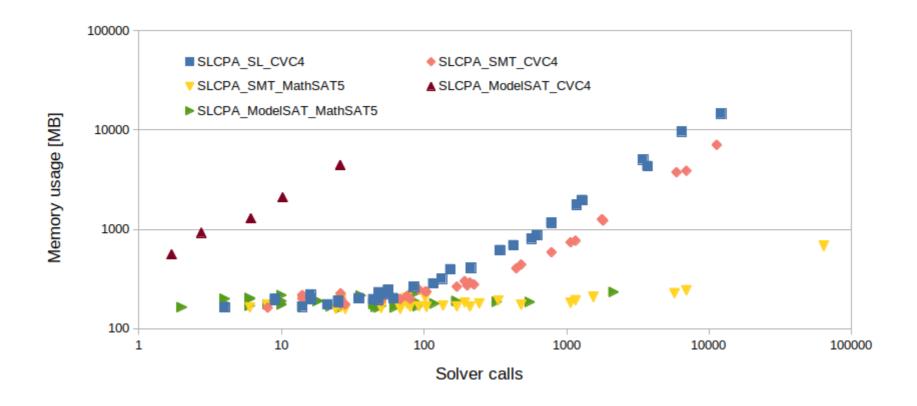
- SLCPA_SL_CVC4 slow and has one timeout
- All SMT aproaches significantly faster
- SLCPA_ModelSAT_MathSAT5 comparable to SMGCPA

Evaluation - Memory



- CVC4 without ModelSAT is memory consuming
- Again SLCPA_ModelSAT_MathSAT5 comparable to SMGCPA

Evaluation - Solver calls and Memory



• CVC4: significant increase of memory consumption with the amount of solver calls; not observable for MathSAT5

 \rightarrow potential memory leak in CVC4 solver interface

Conclusion

- SL solver interface for allocation check and dereferencing is crucial for performance
- However, model checking approach with symbolic execution and SL worked out
- Problems without spatiality are better suited for SMT
- Combination of SL and SMT is promising in respect to composition and efficiency

References

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- [6] M. Sighireanu et al. SL-COMP: Competition of Solvers for Separation Logic. In Proc. TACAS, pages 116–132. Springer International Publishing, 2019. ISBN 978-3-030-17502-3. https://doi.org/10.1007/978-3-030-17502-3_8.
- P. Muller and T.áš Vojnar. CPAlien: Shape Analyzer for CPAChecker. In Proc. TACAS, pages 395–397. Springer Berlin Heidelberg, 2014. ISBN 978-3-642-54862-8. https://doi.org/10.1007/978-3-642-54862-8_28.



Evaluation

· BenchExec tables:

file:///home/mo/Documents/Thesis/talk/benchexec/SLCPA_ModelSAT_CVC4. results.html

file:///home/mo/Documents/Thesis/talk/benchexec/SLCPA_ModelSAT_MathS AT5.results.html

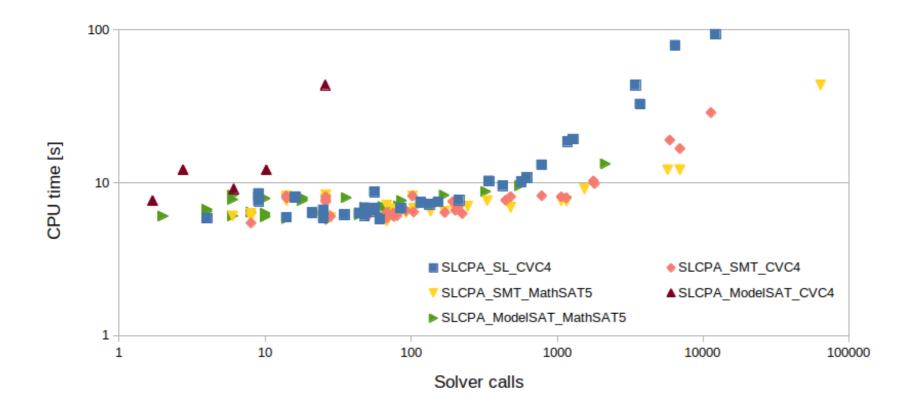
file:///home/mo/Documents/Thesis/talk/benchexec/SLCPA_SL_CVC4.results. html

file:///home/mo/Documents/Thesis/talk/benchexec/SLCPA_SMT_CVC4.html

file:///home/mo/Documents/Thesis/talk/benchexec/SLCPA_SMT_MathSAT5. html

file:///home/mo/Documents/Thesis/talk/benchexec/SMGCPA.results.html

Evaluation - Solver calls and CPU time



- CPU time increases with amount of solver calls
- SLCPA_SMT_CVC4 faster than SLCPA_ModelSAT_CVC4 although significantly more solver calls

Pointer Analysis - Locations and Values

$$\frac{\&x = SymLoc(x)}{\Sigma \models x \Downarrow_{l} \&x} \text{ VAR } \qquad \frac{\Sigma \models \epsilon \Downarrow_{v} E}{\Sigma \models *\epsilon \Downarrow_{l} E} \text{ PTR}$$

$$\frac{\Sigma \models x \Downarrow_{l} E \quad \Sigma \models \epsilon \Downarrow_{v} E' \quad F = E + E' \cdot sizeof(type(*x))}{\Sigma \models x[\epsilon] \Downarrow_{l} F} \text{ ARRAY}$$

$$\frac{\Sigma \models x \Downarrow_{l} E \quad F = E + Offset(x, y)}{\Sigma \models x.y \Downarrow_{l} F} \text{ Field}$$

Operational semantics of \Downarrow_l

Pointer Analysis - Locations and Values

$$\frac{\Sigma \models \kappa \Downarrow_v \kappa}{\Sigma \models \kappa \Downarrow_v \kappa} \operatorname{Const} \quad \frac{\Sigma \models \epsilon_\ell \Downarrow_l E \quad n = sizeof(\epsilon_\ell) \quad F = \Sigma[E]^n}{\Sigma \models \epsilon_\ell \Downarrow_v F} \text{ LHs}$$

$$\frac{\Sigma \models \epsilon_\ell \Downarrow_l E}{\Sigma \models \& \epsilon_\ell \Downarrow_v E} \text{ AddressOr} \quad \frac{\Sigma \models \epsilon_0 \Downarrow_v E \quad G = E \odot F}{\Sigma \models \epsilon_1 \Downarrow_v F \quad type(\epsilon_0) = type(\epsilon_1)}{\Sigma \models \epsilon_0 \odot \epsilon_1 \Downarrow_v G} \text{ BINExP}$$

$$\frac{E = SymVal(f)}{\Sigma \models f(\epsilon_0, ..., \epsilon_n) \Downarrow_v E} \text{ FunCall} \quad \frac{\Sigma \models \epsilon \Downarrow_v E \quad F = cast(E, type(\epsilon), \tau)}{\Sigma \models (\tau) \epsilon \Downarrow_v F} \text{ Cast}$$

 $\frac{\Sigma \models \epsilon_0 \Downarrow_v E}{\Sigma \models \epsilon_1 \Downarrow_v F} \quad \begin{array}{c} \epsilon_0 \text{ is a pointer} \\ G = E \circledast (F \cdot size of(type(*\epsilon_0))) \\ \Sigma \models \epsilon_0 \circledast \epsilon_1 \Downarrow_v G \end{array} \quad PTRARITHMETIC$

Operational semantics of \Downarrow_v

• Declaration edge:

$$\frac{\Sigma \models x \Downarrow_l E \quad n = sizeof(\tau)}{\{\Pi \land \Sigma\} \ \tau \ x \ \{\Pi \land \Sigma \ast E \mapsto -^0, ..., -^{n-1}\}} \text{ Declare}$$

• Statement edge: assignment

$$\begin{split} & \Sigma * E \mapsto -^{0}, ..., -^{n-1} \models \epsilon_{\ell} \Downarrow_{l} E \\ & n = sizeof(\epsilon_{\ell}) \quad \Sigma * E \mapsto -^{0}, ..., -^{n-1} \models \epsilon \Downarrow_{v} F \\ \hline \{\Pi \land \Sigma * E \mapsto -^{0}, ..., -^{n-1}\} \epsilon_{\ell} = \epsilon \ \{\Pi \land \Sigma * E \mapsto F^{0}, ..., F^{n-1}\} \end{split} \text{Assign} \\ & \frac{\Sigma \models \epsilon \Downarrow_{v} nil}{\{\Pi \land \Sigma\} \epsilon_{\ell} = \epsilon \ \{\bot^{R}\}} \text{Assign}^{\text{InvR}} \quad \frac{\Sigma \models \epsilon_{\ell} \Downarrow_{l} nil}{\{\Pi \land \Sigma\} \epsilon_{\ell} = \epsilon \ \{\bot^{W}\}} \text{Assign}^{\text{InvW}} \end{split}$$

• Statement edge: dynamic allocation

$$\begin{split} \overline{\{\Pi \land \Sigma\} \operatorname{malloc}(\epsilon) \ \{\bot^L\}} & \operatorname{Malloc}^{\operatorname{Leak}} \\ & \Sigma \ast E \mapsto -^0, ..., -^{n-1} \models \epsilon_{\ell} \Downarrow_{l} E \qquad n = sizeof(\epsilon_{\ell}) \\ & \Sigma \ast E \mapsto -^0, ..., -^{n-1} \models \epsilon \Downarrow_{v} \kappa \qquad \Sigma \models \operatorname{malloc}(\epsilon) \Downarrow_{v} x \\ & \overline{\{\Pi \land \Sigma \ast E \mapsto -^0, ..., -^{n-1}\} \ \epsilon_{\ell} = \operatorname{malloc}(\epsilon) \ \{ \begin{array}{c} \Pi \land \Sigma \ast E \mapsto x^0, ..., x^{n-1} \\ & \ast x \mapsto -^0, ..., -^{\kappa-1} \end{array} \} \end{split}$$
 MALLOC

- Function call edge:
 - Allocated memory for parameters (Σ^P) and return value (Σ^R)

$$\Sigma^P(f, \{\epsilon_0, \dots, \epsilon_{n-1}\}) := \bigstar_{i=0}^{n-1}(SymLoc(f_i) \mapsto F_i^0, \dots, F_i^{sizeof(\epsilon_i)-1}))$$

$$\Sigma^{R}(f) := SymLoc(f) \mapsto -^{0}, ..., -^{sizeof(f)-1}$$

• Function call edge:

$$\begin{split} & \frac{type(f) = \texttt{void}}{\{\Pi \land \Sigma\} \ f(\epsilon_0, ..., \epsilon_n) \ \{\Pi \land \Sigma \ast \Sigma^P(f, \{\epsilon_0, ..., \epsilon_n\})\}} \ \texttt{FunCall}^{\texttt{void}} \\ & \frac{\Sigma^f = \Sigma^P(f, \{\epsilon_0, ..., \epsilon_n\}) \ast \Sigma^R(f) \ type(f) \neq \texttt{void}}{\{\Pi \land \Sigma\} \ f(\epsilon_0, ..., \epsilon_n) \ \{\Pi \land \Sigma \ast \Sigma^f\}} \ \texttt{FunCall} \\ & \frac{\exists \epsilon \in \{\epsilon_0, ..., \epsilon_n\}. (\Sigma \models \epsilon \Downarrow_v nil)}{\{\Pi \land \Sigma\} \ f(\epsilon_0, ..., \epsilon_n) \ \{\bot^R\}} \ \texttt{FunCall}^{\texttt{Inv}} \end{split}$$

• Return statement edge:

$$\begin{array}{l} \displaystyle \frac{type(f)=\texttt{void}}{\{\Pi\wedge\Sigma\}\texttt{ return}_f\ \{\Pi\wedge\Sigma\}}\texttt{ RETURN}^{\texttt{void}}\\ \\ \displaystyle \frac{E=SymLoc(f)\quad\Sigma\models\epsilon\Downarrow_vF\quad type(f)\neq\texttt{void}\quad n=sizeof(f)}{\{\Pi\wedge\Sigma\ast E\mapsto-^0,...,-^{n-1}\}\texttt{ return}_f\ \epsilon\ \{\Pi\wedge\Sigma\ast E\mapsto F^0,...,F^{n-1}\texttt{ RETURN}} \end{array}$$

• Function return edge:

$$\frac{type(f) = \text{void}}{\{\Pi \land \Sigma * \Sigma^{P}(f, \{x_{0}, ..., x_{n}\})\} f(x_{0}, ..., x_{n}) \{\Pi \land \Sigma\}} \text{ FunRet}^{\text{void}}}$$
$$\frac{type(f) \neq \text{void}}{\{\Pi \land \Sigma * \Sigma^{P}(f, \{x_{0}, ..., x_{n}\}) * \Sigma^{R}(f)\} f(x_{0}, ..., x_{n}) \{\Pi \land \Sigma\}} \text{ FunRet}$$

• Variable Scope:

$$\begin{split} \Sigma &\models x \Downarrow_{l} E \qquad n = sizeof(x) \\ \Sigma &\models x \Downarrow_{v} F \qquad \bigwedge_{i=0}^{n-1} Leak(\Sigma, F^{i}) = true \\ \overline{\{\Pi \land \Sigma \ast E \mapsto F^{0}, ... F^{n-1}\} oos(x) \{\bot^{L}\}} \\ \end{split}$$
OUTOFSCOPE^{Leak}

 special statement oos(x) representing a variable x that goes out of scope

SL State

 $\begin{array}{l} \Sigma^s: \texttt{LinkedHashMap}\langle \texttt{BVFormula} \rightarrow \texttt{BVFormula} \rangle \\ \Sigma^h: \texttt{LinkedHashMap}\langle \texttt{BVFormula} \rightarrow \texttt{BVFormula} \rangle \\ \Pi: \texttt{BooleanFormula} \\ SegmentSizes: \texttt{Map}\langle \texttt{BVFormula} \rightarrow \texttt{Int} \rangle \\ Allocas: \texttt{Map}\langle \texttt{String} \rightarrow \texttt{Set}\langle \texttt{BVFormula} \rangle \rangle \\ SSA-Indices: \texttt{SSAMap} \\ Properties: \texttt{Set}\langle \texttt{String} \rangle \end{array}$

Why SSAMap?

```
int main() {
1
              char *p;
2
              for (char i=0; i<2; i++) {
3
                 char x;
4
                 if(i==0) \{
5
                 p = \&x;
6
                } else {
 7
                   *p = 1;
8
                }
9
                                       \rightarrow SymLoc(x) = &main:x@i
               }
10
              return 0;
11
            }
12
13
```

• Dereferencing with model generation:

```
Input: an expression E as BVFormula and a Map M describing a
       symbolic heap
Output: a pair of BVFormula denoting the location and value of the
         cell at address E if allocated, nil otherwise
if M.containsKey(E) then
   return M.get(E);
end
// Assume auxiliary variable aux_k for each k
formula = \bigwedge_{k \in M.keys()} k = E \iff aux_k;
if SAT(formula) then
   model = getModel(formula);
   foreach (aux_k, value) \in model do
      if value then
          return k;
      end
   end
end
return nil;
```