Interpolation and SAT-Based Model Checking Revisited: Adoption to Software Verification

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Joint work with Dirk Beyer and Nian-Ze Lee

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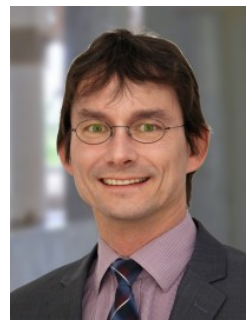

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

CPA

SoSy-Lab Software Systems

Leveraging Knowledge from Related Fields

- Software verification is hard
- Successfully adopted algorithms from hardware
  - BMC [9, 12]
  - k-Induction [17, 4]
  - IC3/PDR [11, 3]

Interpolation and SAT-Based Model Checking

- K. L. McMillan, CAV 2003 [18]
- Interpolation-based model checking (IMC)
  - Finite-state transition systems (circuit)
  - Fixed points with interpolants from unsatisfiable BMC

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- State of the art for hardware

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- State of the art for hardware
- IMC itself has not been used to verify programs

Research Questions

- Efficient adoption of IMC to verify programs
- Performance of IMC against state of the art

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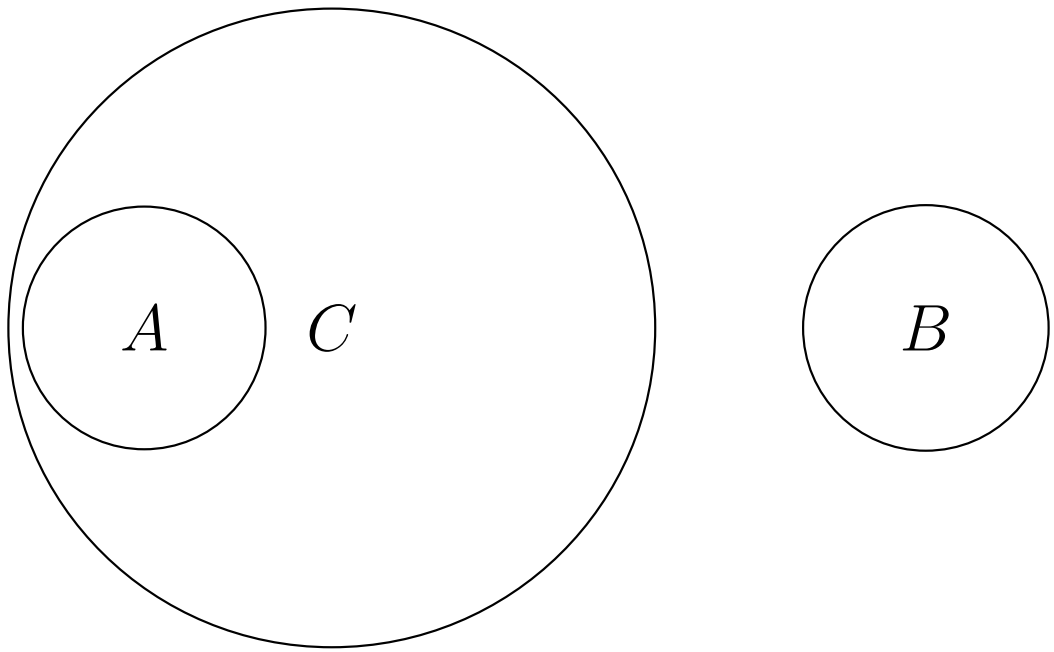
- Performance of IMC against state of the art

Spoiler:
Competitive among polished software-verification methods

Craig Interpolation

- $A(X,Y) \wedge B(Y,Z)$ UNSAT: interpolant $C(Y)$
  - $A(X,Y) \rightarrow C(Y)$
  - $C(Y) \wedge B(Y,Z)$ UNSAT



Interpolation-Based Model Checking

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- Interpolant $C_1(s_1)$: 1-step overapproximation

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- Interpolant $C_1(s_1)$: 1-step overapproximation
- $\underbrace{C_1(s_0)T(s_0, s_1)}_{A'(s_0,s_1)} \underbrace{T(s_1, s_2) \dots T(s_{k-1}, s_k)\neg P(s_k)}_{B'(s_1,s_2,\dots,s_k)}$
  - Interpolant $C_2(s_1)$: 2-step overapproximation
  - Repeat until $\bigvee C_i$ becomes a fixed point
  - Increment k if query becomes satisfiable

Towards an Efficient Adoption

- System under verification $\rightarrow I(s), T(s, s'), P(s)$
  - Sequential circuit: monolithic loop
  - Program: arbitrary control flow

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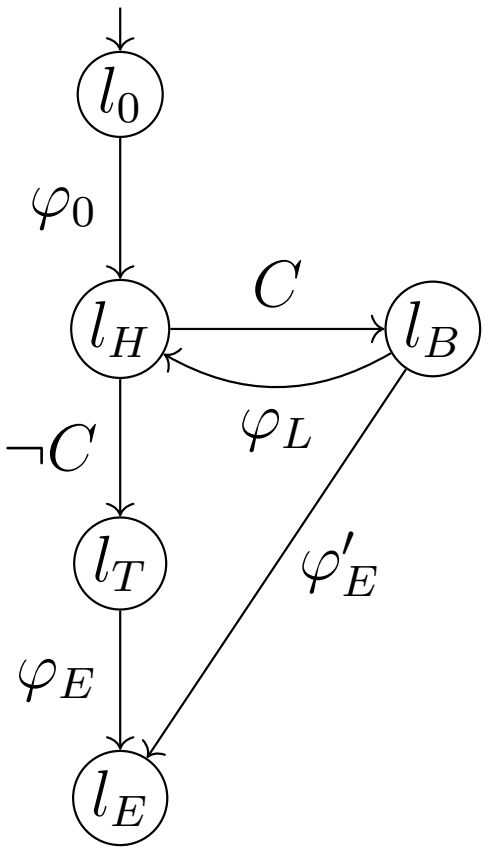
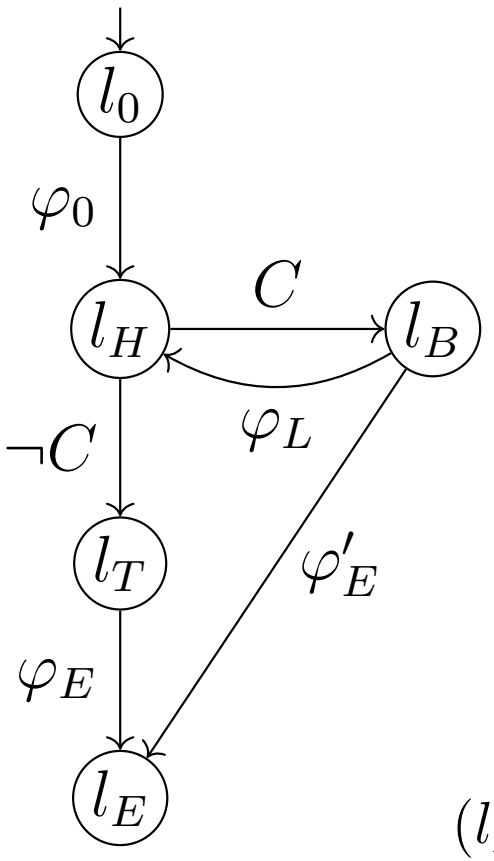
- System under verification $\rightarrow I(s), T(s, s'), P(s)$
  - Sequential circuit: monolithic loop
  - Program: arbitrary control flow
- Challenge: minimal disruption of program structure
- Idea: Use large-block encoding (LBE) [2, 7] to summarize control-flow automaton (CFA)
  - Loop-free blocks replaced by single transitions

Single-Loop CFA summarized by LBE



Single-Loop CFA summarized by LBE



$$(l_0, true)$$
$$\downarrow$$
$$(l_H, \varphi_0|_{i=0})$$
$$\downarrow$$
$$(l_H, (C \wedge \varphi_L)|_{i=1})$$
$$\downarrow$$
$$(l_H, (C \wedge \varphi_L)|_{i=2})$$
$$\vdots$$
$$(l_H, (C \wedge \varphi_L)|_{i=k})$$
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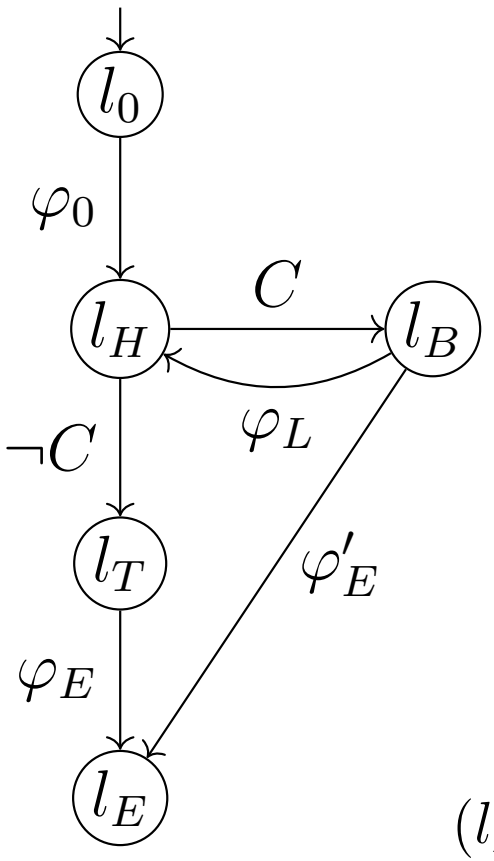
$$(l_E, ((\neg C \wedge \varphi_E) \vee (C \wedge \varphi'_E))|_{i=k})$$

Single-Loop CFA summarized by LBE



$$
\begin{aligned}
&(l_0, \mathit{true}) \\
&\downarrow \\
&(l_H, \varphi_0|_{i=0}) && : I(s_0) \\
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&(l_H, (C \wedge \varphi_L)|_{i=1}) && : T(s_0, s_1) \\
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&\vdots \\
&(l_H, (C \wedge \varphi_L)|_{i=k}) && : T(s_{k-1}, s_k) \\
&\downarrow \\
&(l_E, ((\neg C \wedge \varphi_E) \vee (C \wedge \varphi'_E))|_{i=k}) && : \neg P(s_k)
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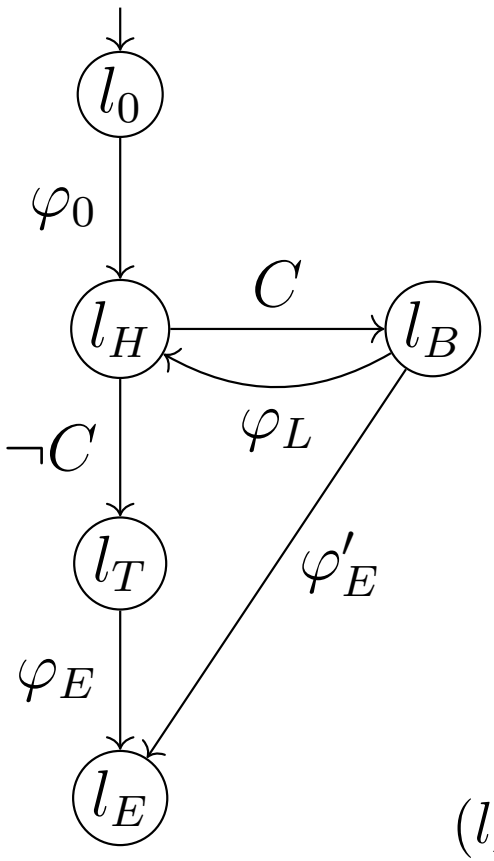
Single-Loop CFA summarized by LBE



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\end{aligned}
$$

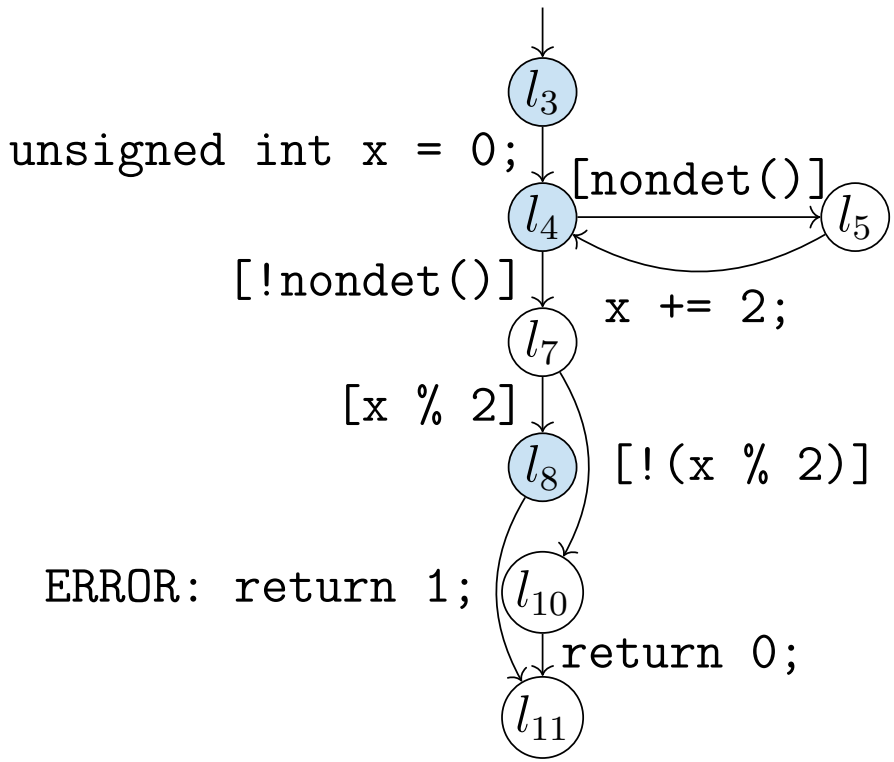
Solution for multi-loop programs:
standard transformation to single loop

Interpolation-based model checking for Software

- Expressed on top of existing unifying framework for SMT-based software verification [5]
  - Provides LBE, CFA unrolling, formula collection
  - Easy comparison with other approaches (k-induction, IMPACT, ...)
  - Details in paper [8]
    `https://www.sosy-lab.org/research/cpa-imc/`
- Implemented in CPACHECKER [6]

Reachability Analysis of Errors

```
1  extern int nondet();
2  int main(void) {
3    unsigned int x = 0;
4    while (nondet()) {
5      x += 2;
6    }
7    if (x % 2) {
8      ERROR: return 1;
9    }
10   return 0;
11 }
```



Example for One Loop Unrolling

$e\colon (l, \varphi)$

$e_0\colon (l_3, \top)$

↓

$e_1\colon (l_4, x_0 = 0)$

↓

$e_5\colon (l_5, \neg(ret_0 = 0))$

↓

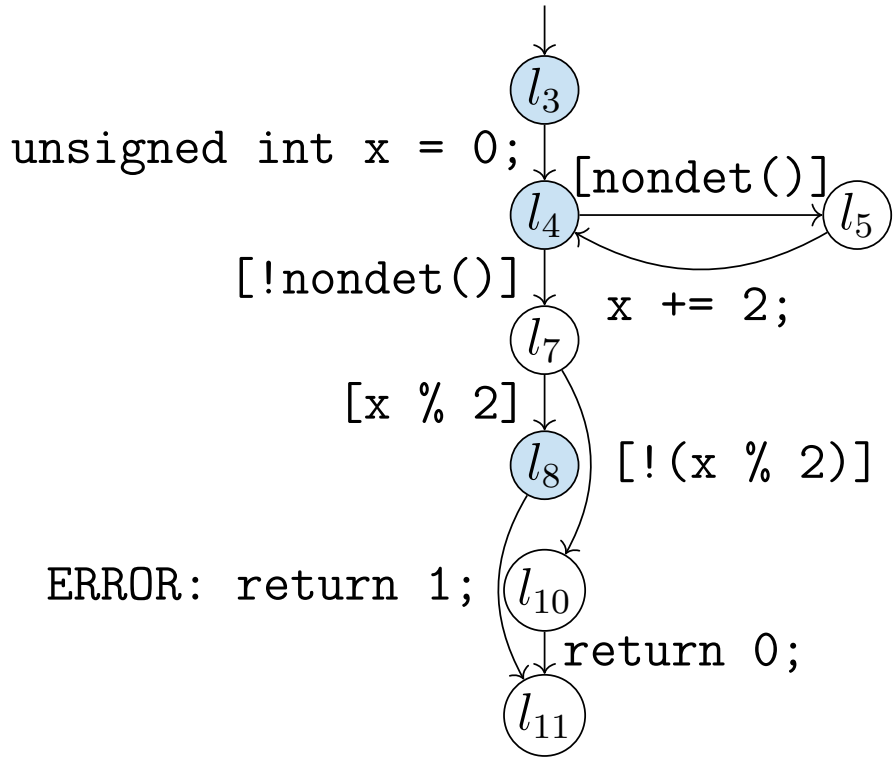
$e_6\colon (l_4, \neg(ret_0 = 0) \wedge x_1 = x_0 + 2)$

↓

$e_7\colon (l_7, ret_1 = 0)$

↓

$e_8\colon (l_8, ret_1 = 0 \wedge \neg(x_1 \% 2 = 0))$



(path to error location l_8 with one loop unrolling)

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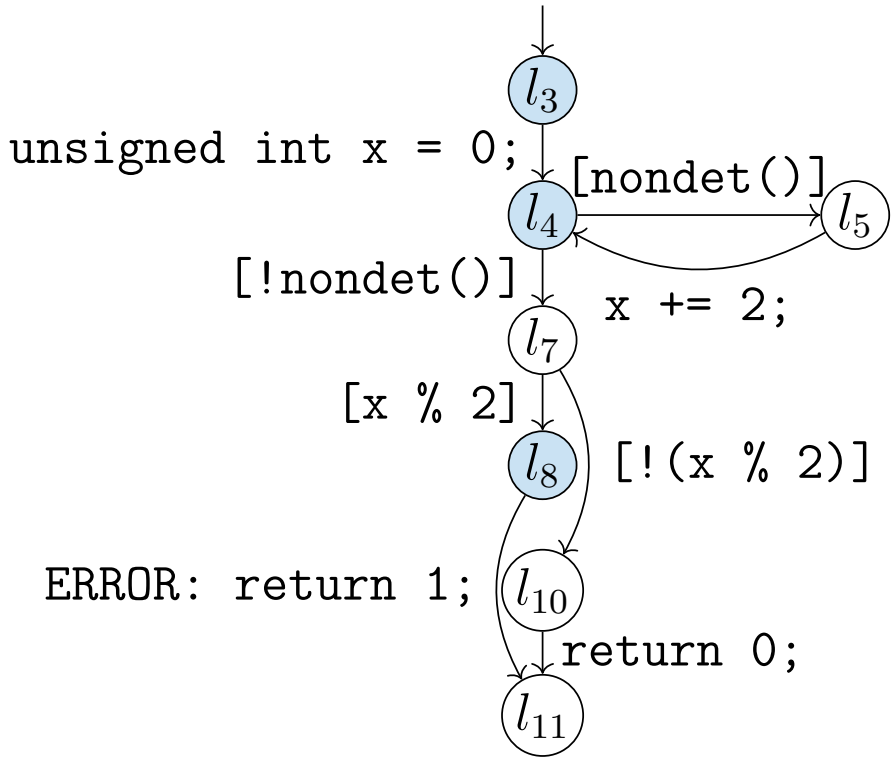

$$\underbrace{x_0 = 0}_{I(s_0) \text{ from } e_1} \wedge \underbrace{\neg(ret_0 = 0) \wedge x_1 = x_0 + 2}_{T(s_0, s_1) \text{ from } e_6} \wedge \underbrace{ret_1 = 0 \wedge \neg(x_1 \% 2 = 0)}_{\neg P(s_1) \text{ from } e_8}$$

$$x_1 \% 2 = 0$$

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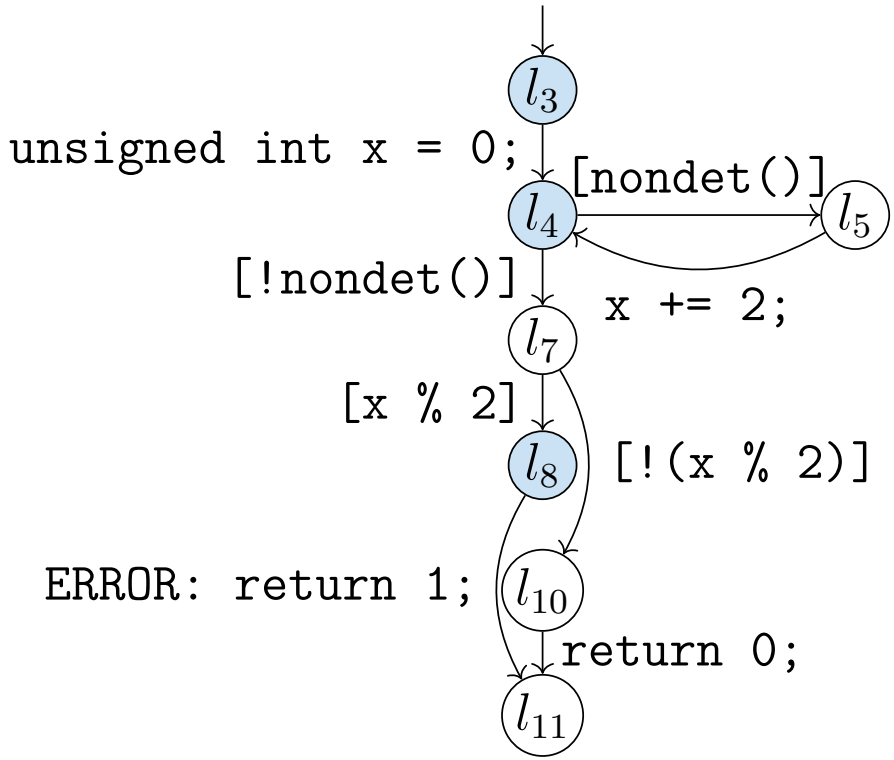

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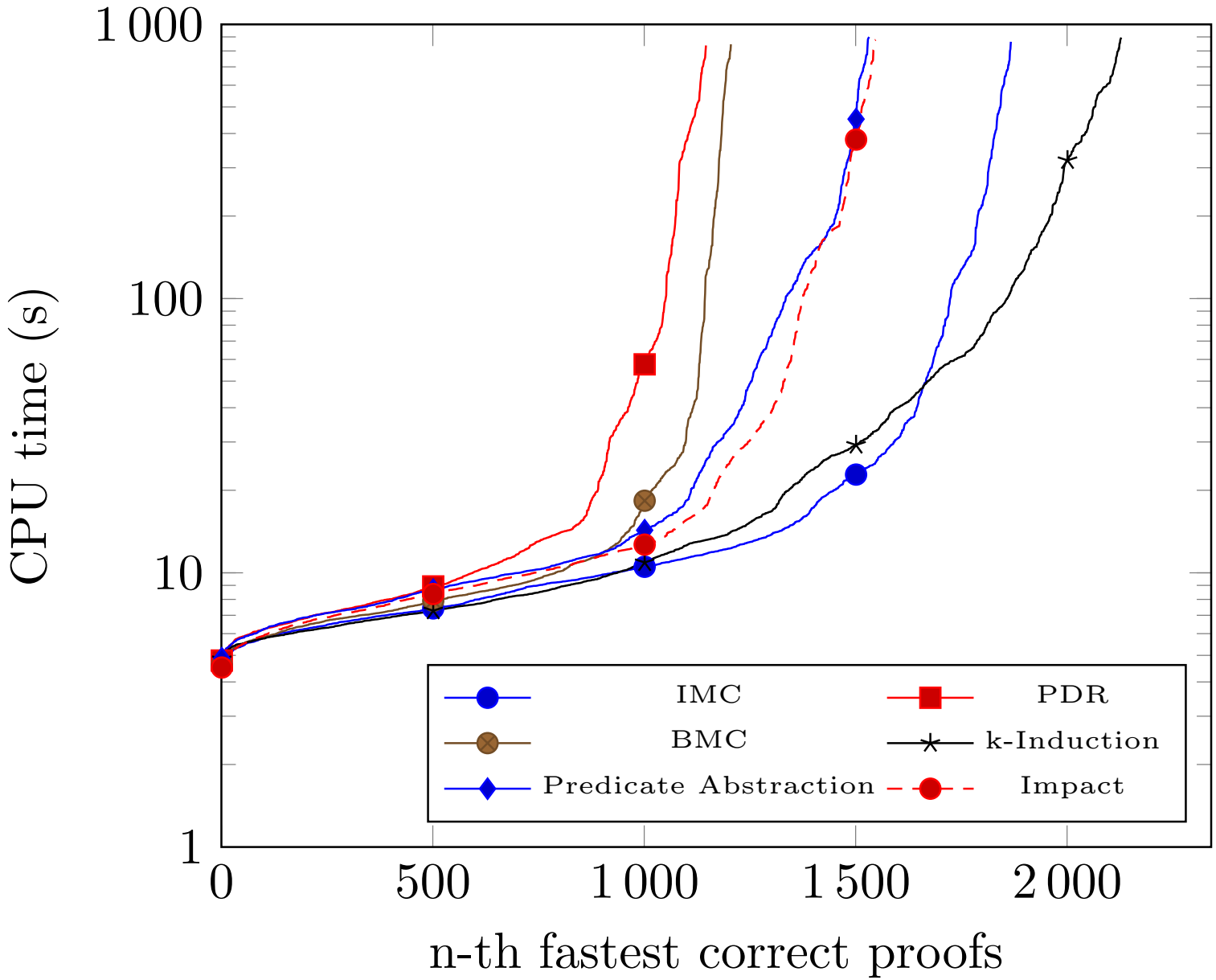
$$\underbrace{x_0 \% 2 = 0}_{C_1(s_0)} \wedge \underbrace{\neg(ret_0 = 0) \wedge x_1 = x_0 + 2}_{T(s_0, s_1) \text{ from } e_6} \wedge \underbrace{ret_1 = 0 \wedge \neg(x_1 \% 2 = 0)}_{\neg P(s_1) \text{ from } e_8}$$

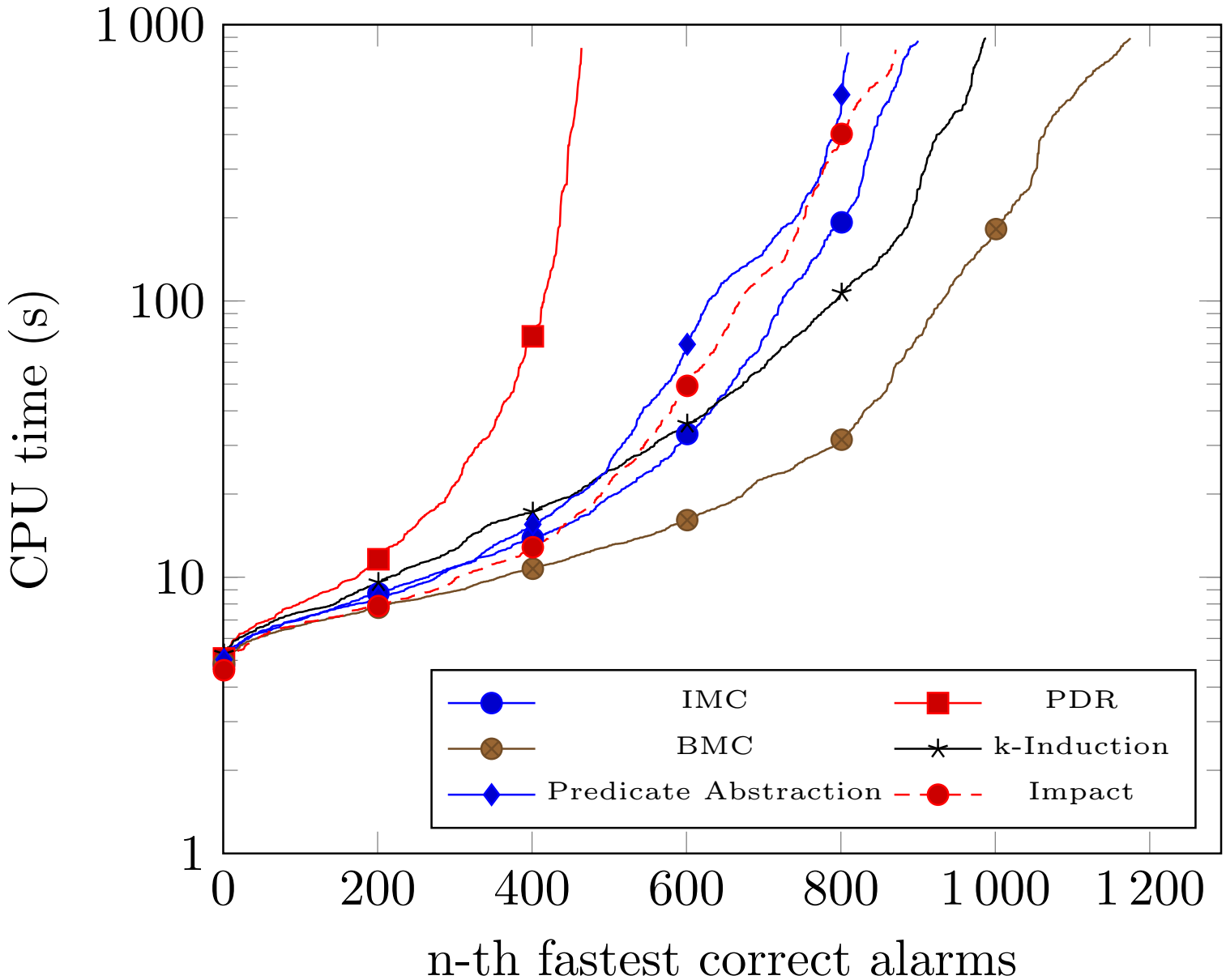
Experimental Setup

- CPAchecker revision 40806
- Interpolants provided by MathSAT5
- Compared algorithms
  - IMC
  - PDR
  - BMC
  - k-Induction
  - Predicate abstraction
  - Impact
- Subset of *ReachSafety* from SV-COMP '22 [1]
  - Safe: 4234 tasks
  - Unsafe: 1793 tasks

Quantile Plot: Safe Tasks



Quantile Plot: Unsafe Tasks



Conclusion

- RQ1: Efficient adoption achieved by large-block encoding
- RQ2: Most efficient and effective interpolation-based algorithm in our evaluation
- Successful adoption of IMC to software
- 19 year old knowledge gap closed

https://www.sosy-lab.org/research/cpa-imc/ [8]

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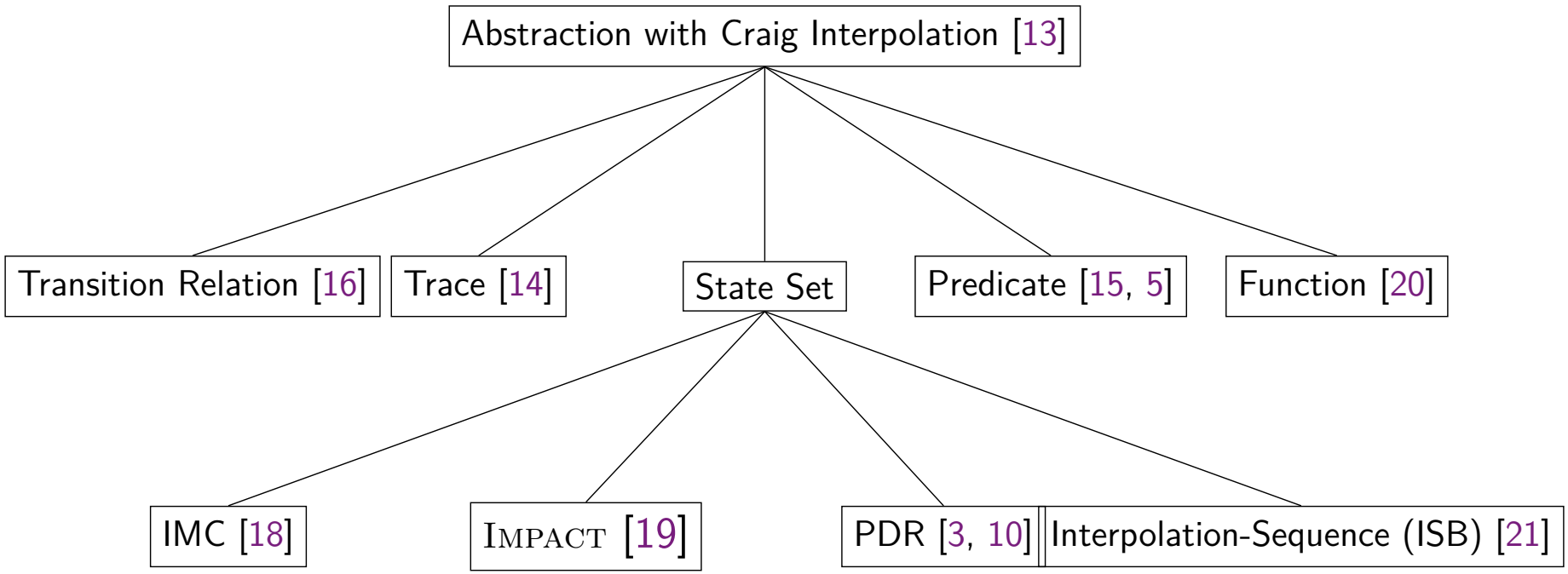
Interpolation-Based Verification Approaches



IMC: Main Procedure

Input: $\mathbb{D} = \mathbb{L} \times \mathbb{P} \times \mathbb{LB}$ and k_{max}
Output: **false** if l_E reachable; **true** if fixed point obtained;
unknown otherwise

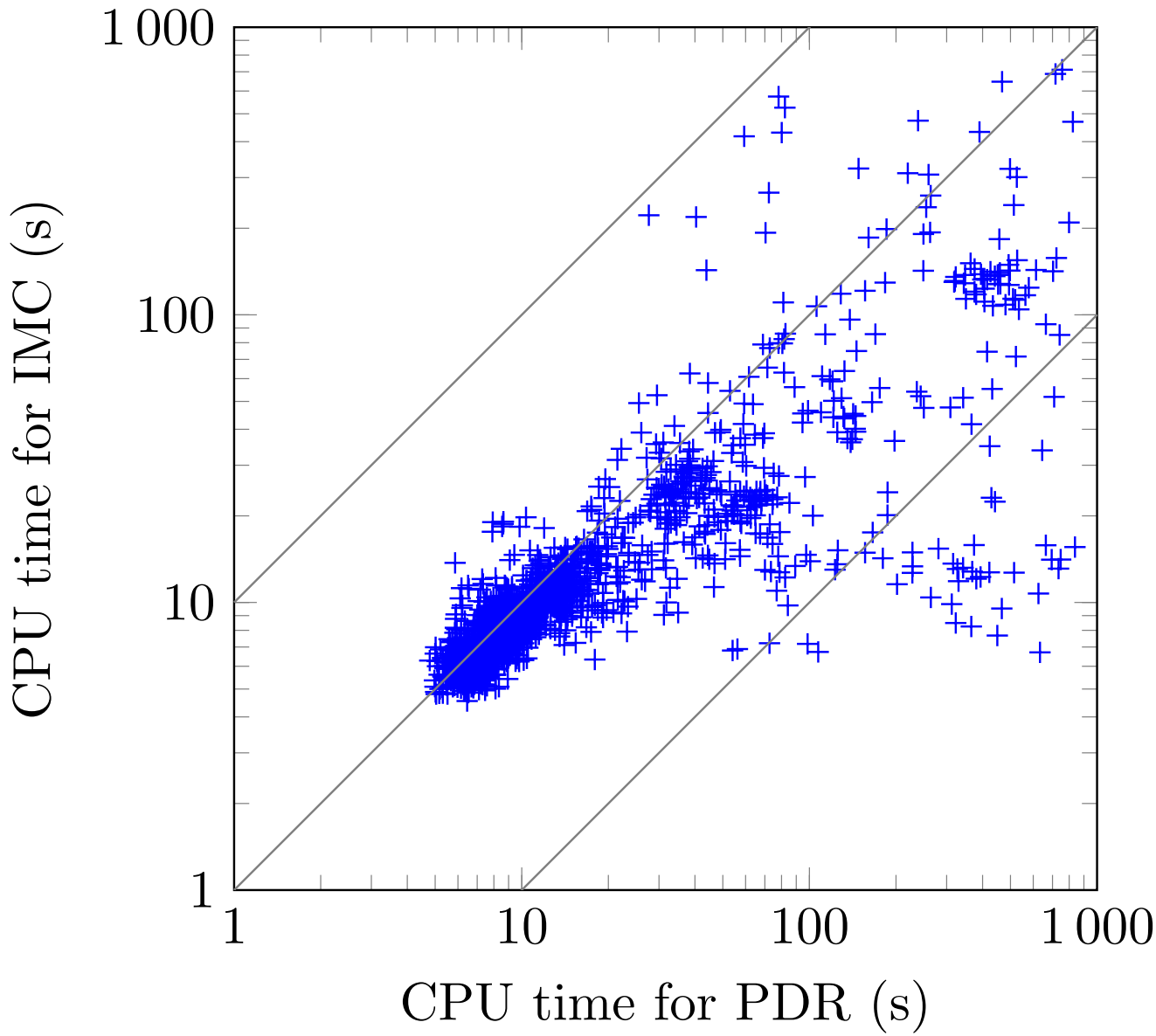
1: $k := 1$
2: $e_0 := (l_0, (true, l_0, true, true), \{l_H \mapsto -1\})$
3: reached := waitlist := $\{e_0\}$
4: **while** $k \leq k_{max}$ **do**
5: (reached, waitlist) := CPA++($\mathbb{D}$, reached, waitlist, k)
6: $(\sigma_p, \sigma_l, \sigma_s)$:= collect_formulas(reached, k)
7: **if** sat($\sigma_p \wedge \sigma_l \wedge \sigma_s$) **then**
8: **return false**
9: **if** $k > 1$ **and** reach_fixed_point(σ_p,σ_l,σ_s) **then**
10: **return true**
11: $k := k + 1$
12: **return unknown**

IMC: reach_fixed_point(σ_p,σ_l,σ_s)

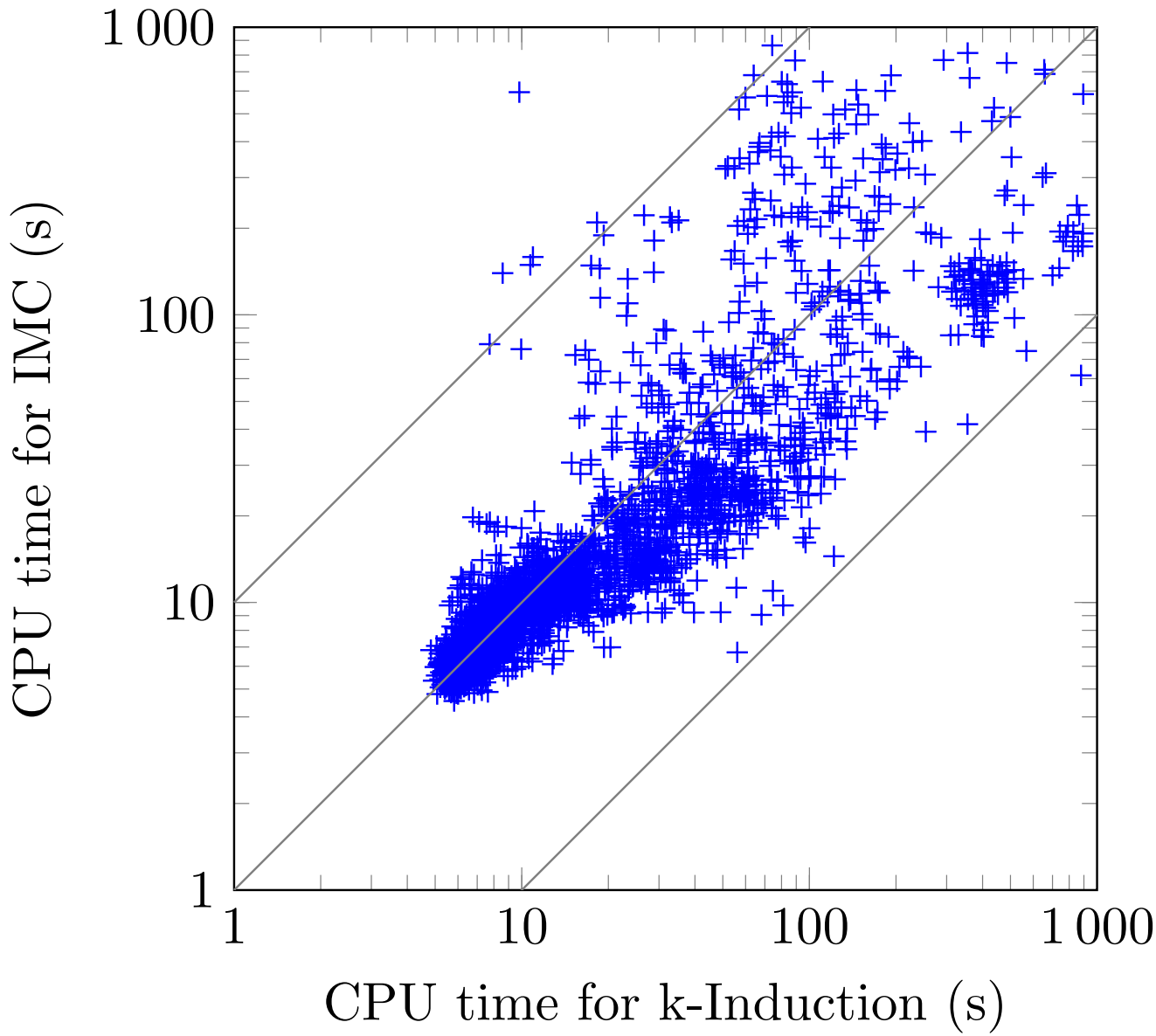
Input: σ_p, σ_l, and σ_s
Output: **true** if fixed point obtained; **false** otherwise

```
1: image := start := σ_p
2: while ¬sat(start ∧ σ_l ∧ σ_s) do
3:     τ := get_interpolant(start ∧ σ_l, σ_s)
4:     τ := shift_variable_index(τ, σ_p)
5:     if ¬sat(τ ∧ ¬image) then
6:         return true
7:     image := image ∨ τ
8:     start := τ
9: return false
```

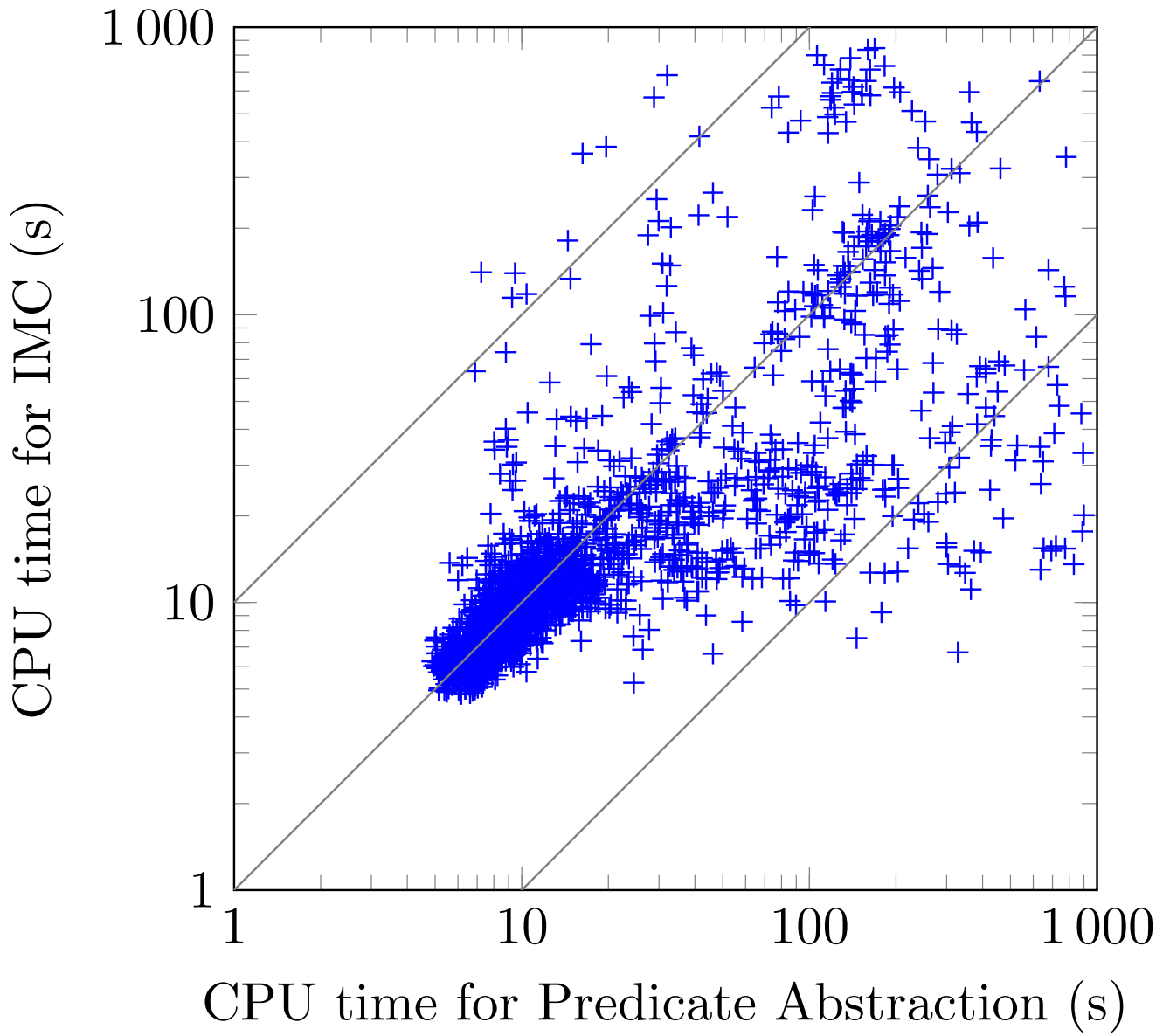
CPU-Time Scatter Plot: vs. PDR



CPU-Time Scatter Plot: vs. k-Induction



CPU-Time Scatter Plot: vs. Predicate Abstraction



CPU-Time Scatter Plot: vs. Impact