Augmenting Interpolation-Based Model Checking with Auxiliary Invariants

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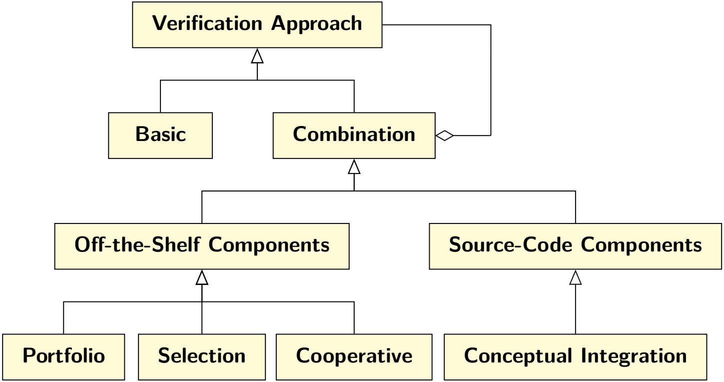
COOP 2023-04-23

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

SoSy-Lab Software Systems

Cooperative Approaches



(Figure taken from Dirk Beyer's keynote talk at FTSCS 2022)

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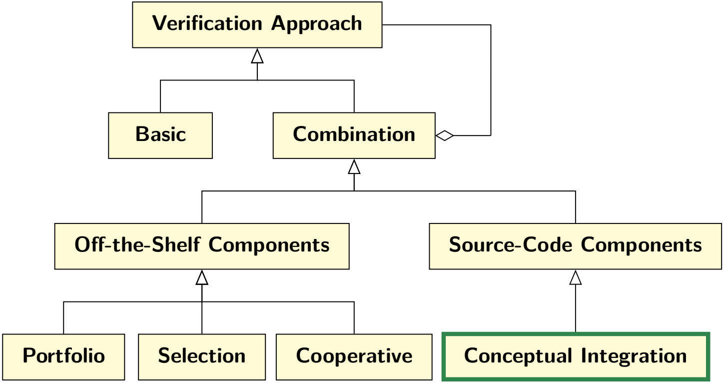

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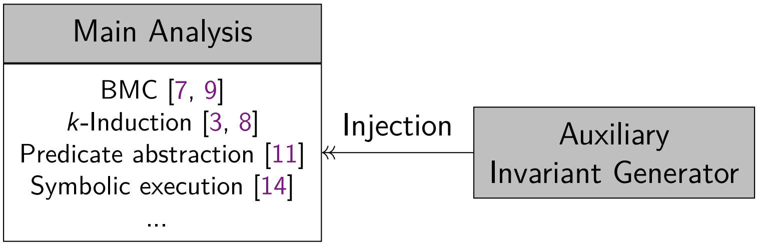
Cooperation via Invariant Injection



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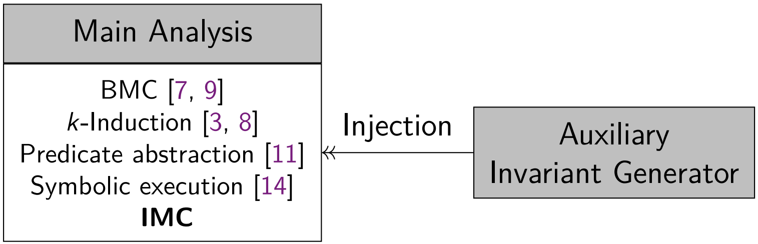

Interpolation and SAT-Based Model Checking

- K. L. McMillan, CAV 2003 [12]
- Interpolation-based model checking (IMC)
  - Originally designed for finite-state transition systems
  - Compute fixed points with interpolants derived from unsatisfiable BMC queries

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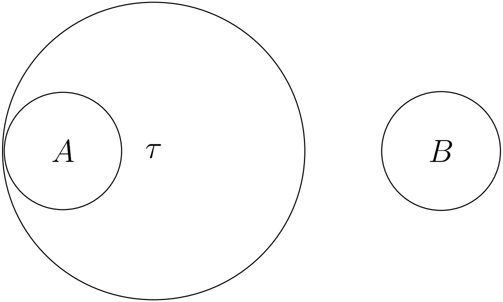
- K. L. McMillan, CAV 2003 [12]
- Interpolation-based model checking (IMC)
  - Originally designed for finite-state transition systems
  - Compute fixed points with interpolants derived from unsatisfiable BMC queries
- State of the art for hardware verification
- Recently adopted for verifying software programs [6]

Craig Interpolation

- If $A(X, Y) \wedge B(Y, Z)$ is UNSAT: interpolant $\tau(Y)$
  - $A(X, Y) \Rightarrow \tau(Y)$
  - $\tau(Y) \wedge B(Y, Z)$ is UNSAT



Interpolation-Based Model Checking

- State-transition system: $I(s), T(s, s'), P(s)$

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- $I(s_0)T(s_0, s_1)\ T(s_1, s_2) \ldots T(s_{k-1}, s_k)\neg P(s_k)$

Interpolation-Based Model Checking

- State-transition system: $I(s), T(s, s'), P(s)$
- $\underbrace{I(s_0)T(s_0, s_1)}_{A_0(s_0,s_1)} \underbrace{T(s_1, s_2) \dots T(s_{k-1}, s_k) \neg P(s_k)}_{B(s_1,s_2,\dots,s_k)}$
- Interpolant $\tau_1(s_1)$: 1-step overapproximation

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- Interpolant $\tau_1(s_1)$: 1-step overapproximation
- $\underbrace{\tau_1(s_0)T(s_0, s_1)}_{A_1(s_0,s_1)} \underbrace{T(s_1, s_2) \ldots T(s_{k-1}, s_k) \neg P(s_k)}_{B(s_1,s_2,\ldots,s_k)}$
  - Interpolant $\tau_2(s_1)$: 2-step overapproximation
  - Repeat until $I \vee \bigvee \tau_i$ becomes a fixed point
  - Increment k if a query becomes satisfiable

Strengthen Interpolants with Auxililiary Invariants

- Given an **inductive** invariant Inv, interpolant τ_i can be strengthened by

$$\tau_i' = \tau_i \land Inv$$

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Strengthen Interpolants with Auxililiary Invariants

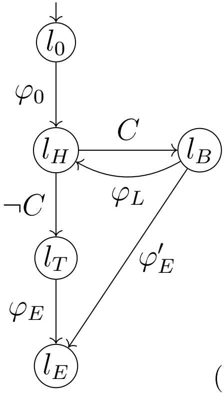
- Given an **inductive** invariant Inv, interpolant τ_i can be strengthened by

$$\tau_i' = \tau_i \wedge Inv$$

- τ_i' is a valid interpolant for IMC
- Inv helps remove some **unreachable** states in τ_i
- Adding more constraints $\rightarrow$ less likely to become SAT

IMC for Software Verification

- Summarize single-loop CFA by LBE [2, 6]



$$(l_0, \mathit{true})$$
$$\downarrow$$
$$(l_H, \varphi_0|_{i=0}) \;:\; I(s_0)$$
$$\downarrow$$
$$(l_H, (C \wedge \varphi_L)|_{i=1}) \;:\; T(s_0, s_1)$$
$$\downarrow$$
$$(l_H, (C \wedge \varphi_L)|_{i=2}) \;:\; T(s_1, s_2)$$
$$\vdots$$
$$(l_H, (C \wedge \varphi_L)|_{i=k}) \;:\; T(s_{k-1}, s_k)$$
$$\downarrow$$
$$(l_E, ((\neg C \wedge \varphi_E) \vee (C \wedge \varphi'_E))|_{i=k}) \;:\; \neg P(s_k)$$

(For multi-loop programs: standard transformation to single loop)

Collecting Formulas

```
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

- $s = \{x, j\}$

Collecting Formulas

I

```
int main(void) {
  unsigned x = 0;
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    x += 2;
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```

- $s = \{x, j\}$
- $I(s)$: $(x = 0) \land (j = 0)$

Collecting Formulas

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
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    j += 1;
    if (j == 2)
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  }
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  return 0;
}
```

- $s = \{x, j\}$
- $I(s)$: $(x = 0) \land (j = 0)$
- $T(s_i, s_{i+1})$:

$$
\begin{aligned}
&(x'_i = x_i + 2) \\
&\land(j_i = 3 \Rightarrow x_{i+1} = x'_i + 1) \\
&\land(j_i \neq 3 \Rightarrow x_{i+1} = x'_i) \\
&\land(j'_i = j_i + 1) \\
&\land(j'_i = 2 \Rightarrow j_{i+1} = 0) \\
&\land(j'_i \neq 2 \Rightarrow j_{i+1} = j'_i)
\end{aligned}
$$

Collecting Formulas

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  int main(void) {
I   unsigned x = 0;
    unsigned j = 0;
    while (nondet()) {
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      j += 1;
      if (j == 2)
        j = 0;
    }
    if (x % 2) {
P     ERROR: return 1;
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    return 0;
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- $s = \{x, j\}$
- $I(s)$: $(x = 0) \land (j = 0)$
- $T(s_i, s_{i+1})$:
  $$(x'_i = x_i + 2)$$
  $$\land(j_i = 3 \Rightarrow x_{i+1} = x'_i + 1)$$
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  $$\land(j'_i = j_i + 1)$$
  $$\land(j'_i = 2 \Rightarrow j_{i+1} = 0)$$
  $$\land(j'_i \neq 2 \Rightarrow j_{i+1} = j'_i)$$
- $P(s)$: $x\%2 = 0$

Plain IMC Example Run

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
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}
```

One loop unrolling ($k = 1$)

- $I \wedge T \wedge \neg P$ is UNSAT
- τ_1: $x \% 2 = 0$

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     return 0;
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One loop unrolling ($k = 1$)

- $I \land T \land \neg P$ is UNSAT
- τ_1: $x\%2 = 0$
- $\tau_1 \land T \land \neg P$ is SAT
- Increase k

Augmented IMC Example Run

```c
int main(void) {
```

I

```c
  unsigned x = 0;
  unsigned j = 0;
```

T

```c
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
```

P

```c
  if (x % 2) {
    ERROR: return 1;
  }
```

```c
  return 0;
}
```

One loop unrolling ($k = 1$)

- Inv: $0 \leq j \leq 1$

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```
   int main(void) {
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- τ_1: $x\%2 = 0$
- τ_1': $\tau_1 \wedge Inv$
- $\tau_1' \wedge T \wedge \neg P$ is UNSAT

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- τ_1: $x\%2 = 0$
- τ_1': $\tau_1 \wedge Inv$
- $\tau_1' \wedge T \wedge \neg P$ is UNSAT
- τ_2: $x\%2 = 0$
- τ_2': $\tau_2 \wedge Inv$

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    int main(void) {
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- τ_1': $\tau_1 \wedge Inv$
- $\tau_1' \wedge T \wedge \neg P$ is UNSAT
- τ_2: $x\%2 = 0$
- τ_2': $\tau_2 \wedge Inv$
- $\tau_2' \equiv \tau_1'$: fixed point!

Evaluation

We conducted experiments to answer the following research questions

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  - Reduce #unrollings and #interpolation-queries?
    Yes, there are cases with >10x improvement
  - Boost run-time efficiency?
    Yes, especially the walltime for solving harder tasks
- **RQ2**: Is the augmented IMC competitive?
  Yes, more proofs compared to other SMT-based algorithms

Invariant Generator

- Continuously-refining data-flow analysis (DF) based on intervals [3, 4]
- Invariants are expressions over intervals
  - e.g. $(0 \leq j \leq 1) \land (x < 5 \lor x > 7)$
- Invariant injection denoted as $\leftarrow\!\circlearrowleft\!-$DF

Tool Configurations and Benchmarks

- CPAchecker[1] revision 42901 of branch *imc-with-invariants*
- Interpolants computed by MathSAT5

[1] https://cpachecker.sosy-lab.org/

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- Compared SMT-based algorithms
  - IMC [6] vs. IMC←↺—DF
  - KI←↺—DF [3], predicate abstraction [10], Impact [13]

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- Compared SMT-based algorithms
  - IMC [6] vs. IMC←↺−DF
  - KI←↺−DF [3], predicate abstraction [10], Impact [13]
- Safe tasks from *ReachSafety* of SV-COMP ’22 [1]
  - Eliminate easy tasks solved by BMC within 900 s
  - 1623 tasks remaining, 870 with non-trivial invariants

[1] https://cpachecker.sosy-lab.org/

Experimental Setup

- Environment
  - OS: Ubuntu 22.04 (64 bit)
  - Machine: 3.4 GHz CPU (8 cores) and 33 GB of RAM
- Each task is limited to
  - 4 CPU cores
  - 900 s of CPU time (max 150 s for DF)
  - 15 GB of RAM

  (reliable resource management by BenchExec[2])

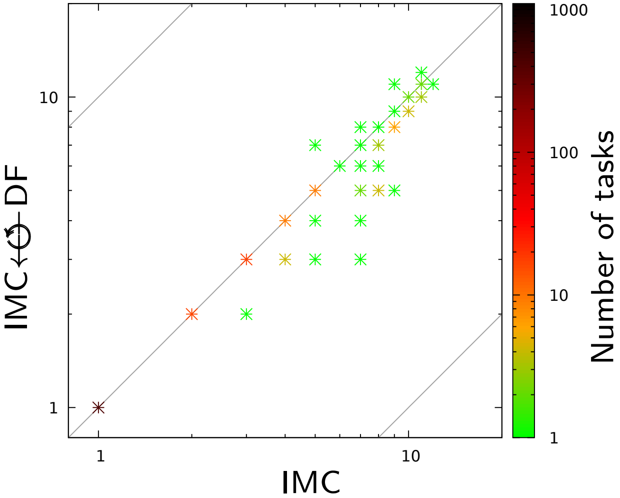
[2] https://github.com/sosy-lab/benchexec

Tasks with Significant Improvement

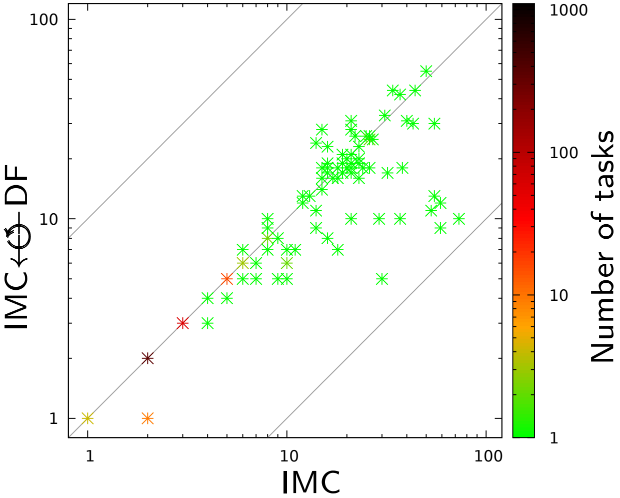
Task	IMC (timeout) #unroll	#itp	walltime	IMC↺DF (solved) #unroll	#itp	walltime
benchmark37_conj	318	316	892	2	2	1.83
s3_srvr_1a.BV.c.cil	65	441	885	6	13	6.26
Problem03_label51	11	57	878	6	11	24.3
Problem03_label15	8	54	876	6	11	35.4
Problem03_label03	9	50	872	6	11	27.2
s3_srvr_2a_alt.BV	38	278	883	20	125	71.7
Problem05_label12	13	42	876	10	32	400
s3_srvr_2a.BV.c.cil	40	290	887	36	252	635
Problem05_label50	13	39	878	11	37	531
Problem05_label07	12	50	873	12	48	699

(time unit: s)

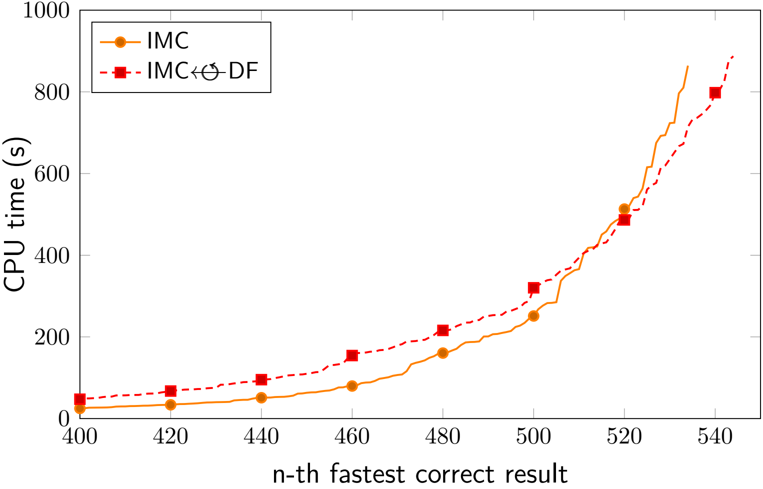
Scatter Plot: #Unrollings



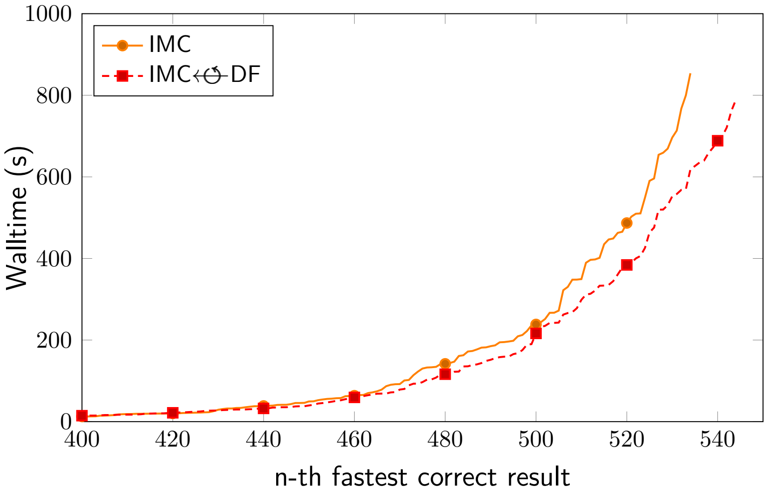
Scatter Plot: #Interpolation-Queries



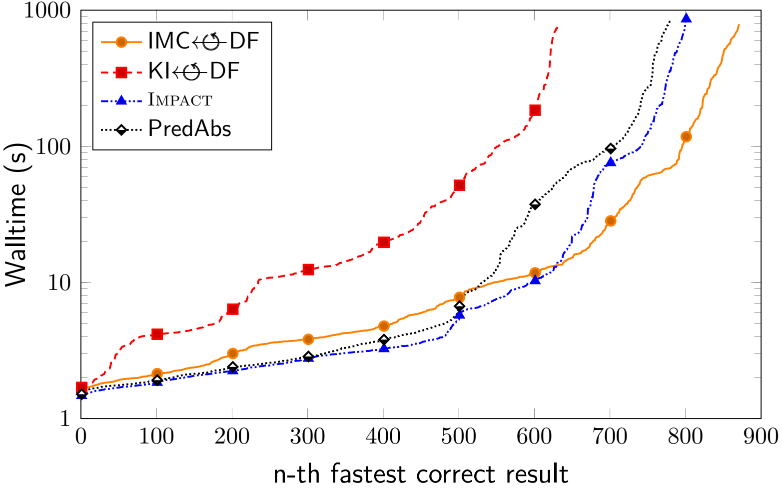
Quantile Plot: CPU time



Quantile Plot: Walltime



Quantile Plot: Comparsion with Others



Conclusion

- Augment IMC via invariant injection
- Open-source implementation in CPAchecker
- In our evaluation, the proposed method can
  - Reduce program unrollings and interpolation queries
  - Improve walltime efficiency
  - Find more proofs

References I

[1] Beyer, D.: Progress on software verification: SV-COMP 2022. In: Proc. TACAS (2). pp. 375–402. LNCS 13244, Springer (2022). https://doi.org/10.1007/978-3-030-99527-0_20

[2] Beyer, D., Cimatti, A., Griggio, A., Keremoglu, M.E., Sebastiani, R.: Software model checking via large-block encoding. In: Proc. FMCAD. pp. 25–32. IEEE (2009). https://doi.org/10.1109/FMCAD.2009.5351147

[3] Beyer, D., Dangl, M., Wendler, P.: Boosting k-induction with continuously-refined invariants. In: Proc. CAV. pp. 622–640. LNCS 9206, Springer (2015). https://doi.org/10.1007/978-3-319-21690-4_42

[4] Beyer, D., Dangl, M., Wendler, P.: Combining k-induction with continuously-refined invariants. Tech. Rep. MIP-1503, University of Passau (January 2015), arXiv:1502.00096

[5] Beyer, D., Keremoglu, M.E., Wendler, P.: Predicate abstraction with adjustable-block encoding. In: Proc. FMCAD. pp. 189–197. FMCAD (2010), https://www.sosy-lab.org/research/pub/2010-FMCAD.Predicate_Abstraction_with_Adjustable-Block_Encoding.pdf

[6] Beyer, D., Lee, N.Z., Wendler, P.: Interpolation and SAT-based model checking revisited: Adoption to software verification. arXiv/CoRR **2208**(05046) (July 2022). https://doi.org/10.48550/arXiv.2208.05046

References II

[7] Cheng, X., Hsiao, M.S.: Simulation-directed invariant mining for software verification. In: Proc. DATE. pp. 682–687. ACM (2008). https://doi.org/10.1109/DATE.2008.4484757

[8] Donaldson, A.F., Haller, L., Kröning, D.: Strengthening induction-based race checking with lightweight static analysis. In: Proc. VMCAI. pp. 169–183. LNCS 6538, Springer (2011). https://doi.org/10.1007/978-3-642-18275-4_13

[9] Ganai, M.K., Gupta, A.: Accelerating high-level bounded model checking. In: Proc. ICCAD. pp. 794–801. ACM (2006). https://doi.org/10.1145/1233501.1233664

[10] Henzinger, T.A., Jhala, R., Majumdar, R., McMillan, K.L.: Abstractions from proofs. In: Proc. POPL. pp. 232–244. ACM (2004). https://doi.org/10.1145/964001.964021

[11] Jain, H., Ivancic, F., Gupta, A., Shlyakhter, I., Wang, C.: Using statically computed invariants inside the predicate abstraction and refinement loop. In: Proc. CAV. pp. 137–151. LNCS 4144, Springer (2006). https://doi.org/10.1007/11817963_15

[12] McMillan, K.L.: Interpolation and SAT-based model checking. In: Proc. CAV. pp. 1–13. LNCS 2725, Springer (2003). https://doi.org/10.1007/978-3-540-45069-6_1

References III

[13] McMillan, K.L.: Lazy abstraction with interpolants. In: Proc. CAV. pp. 123–136. LNCS 4144, Springer (2006). https://doi.org/10.1007/11817963_14

[14] Pasareanu, C.S., Visser, W.: Verification of Java programs using symbolic execution and invariant generation. In: Proc. SPIN. pp. 164–181. LNCS 2989, Springer (2004). https://doi.org/10.1007/978-3-540-24732-6_13

Adopting IMC for Software Verification

- System under verification $\rightarrow I(s), T(s, s'), P(s)$
  - Sequential circuit: monolithic loop
  - Program: arbitrary control flow
- Solution: Use large-block encoding (LBE) [2, 5] to summarize control-flow automaton (CFA)
  - Loop-free blocks replaced by single transitions

IMC: Main Procedure

Input: $\mathbb{D} = \mathbb{L} \times \mathbb{P} \times \mathbb{LB}$ and k_{max}
Output: **false** if l_E reachable; **true** if fixed point obtained; **unknown** otherwise

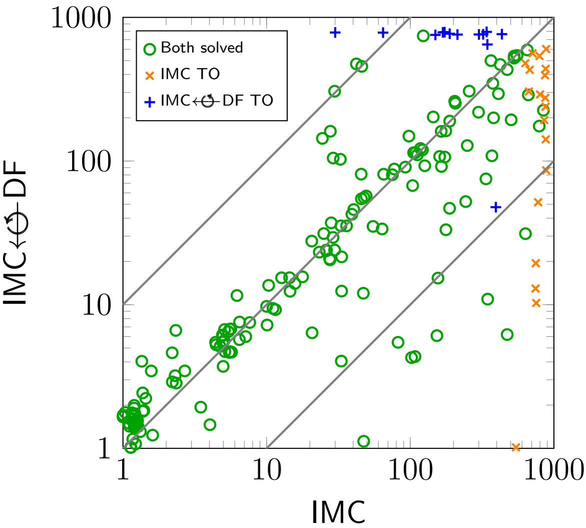
$k := 1$
$e_0 := (l_0, (true, l_0, true, true), \{l_H \mapsto -1\})$
reached := waitlist := $\{e_0\}$
while $k \leq k_{max}$ **do**
    (reached, waitlist) := CPA++($\mathbb{D}$, reached, waitlist, k)
    $(\sigma_p, \sigma_l, \sigma_s)$:= collect_formulas(reached, k)
    if sat($\sigma_p \wedge \sigma_l \wedge \sigma_s$) **then**
        return false
    if $k > 1$ **and** reach_fixed_point(σ_p,σ_l,σ_s) **then**
        return true
    $k := k + 1$
return unknown

IMC: reach_fixed_point(σ_p,σ_l,σ_s)

Input: σ_p, σ_l, and σ_s
Output: **true** if fixed point obtained; **false** otherwise

image := start := σ_p
while $\neg$sat(start $\wedge\ \sigma_l \wedge \sigma_s$) **do**
    τ := get_interpolant(start $\wedge\ \sigma_l, \sigma_s$)
    τ := shift_variable_index(τ, σ_p)
    if $\neg$sat($\tau \wedge \neg$image) **then**
        return true
    image := image $\vee\ \tau$
    start := τ
return false

Scatter Plot: Interpolation-Time



Scatter Plot: Wallime