Augmenting Interpolation-Based Model Checking with Auxiliary Invariants

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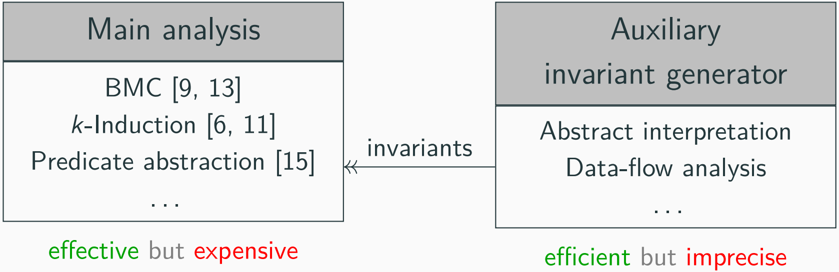
LMU Munich, Germany

Cooperative Verification via Invariant Injection



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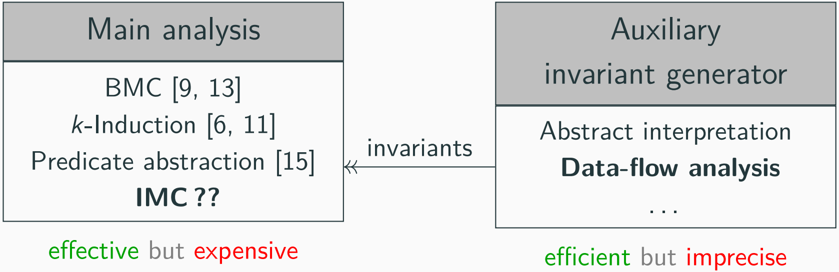
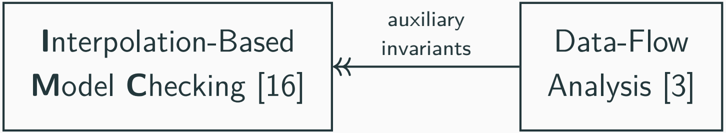


Highlights



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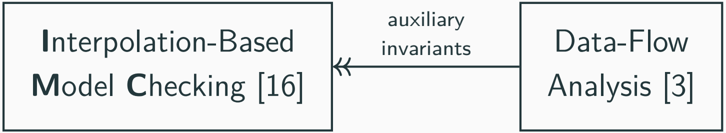

- Novelty: 1st to combine IMC with data-flow analysis
- In our evaluation, augmented IMC
  - was faster and more effective than plain IMC
  - tackled tasks unsolvable by state-of-the-art verifiers

Example Program

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int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
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- ERROR unreachable

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- *fp* is non-inductive

  $\rightarrow$ needs refinement!

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- $x \% 2 = 0 \land 0 \leq j \leq 1$ is an inductive (and safe) invariant!

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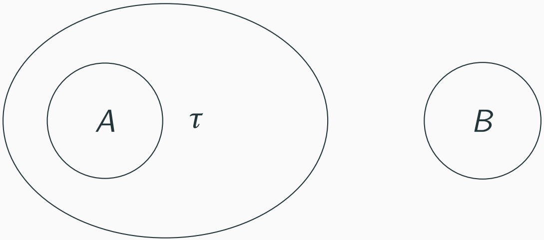
- Interpolation-based model checking (IMC)
  - Originally designed for finite-state transition systems
  - Compute fixed points by interpolating unsatisfiable BMC queries
- State of the art for hardware verification
- Recently adopted for verifying software programs[1]

[1] *Interpolation and SAT-Based Model Checking Revisited: Adoption to Software Verification* (to appear in JAR [7])

Craig Interpolation

- If $A(X,Y) \wedge B(Y,Z)$ is UNSAT: interpolant $\tau(Y)$
  - $A(X,Y) \Rightarrow \tau(Y)$ is valid
  - $\tau(Y) \wedge B(Y,Z)$ is UNSAT



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  - if SAT, counterexample found
  - if UNSAT, enter *interpolation stage*

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  - Interpolant $\tau_2(s_1)$: 2-step safe overapproximation
  - Derive n-step overapproximation τ_n iteratively

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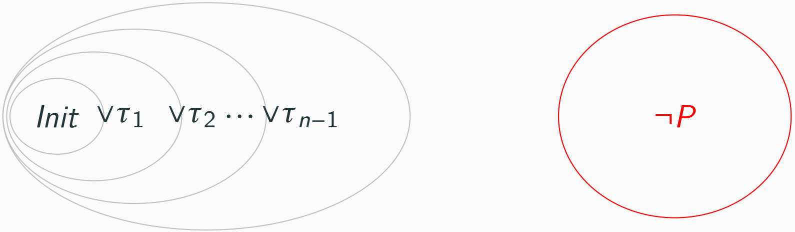

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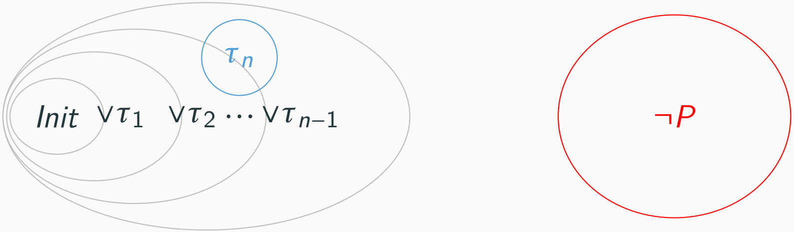
$\tau_n \Rightarrow Init \lor \bigvee_{i=1}^{n-1} \tau_i$ holds



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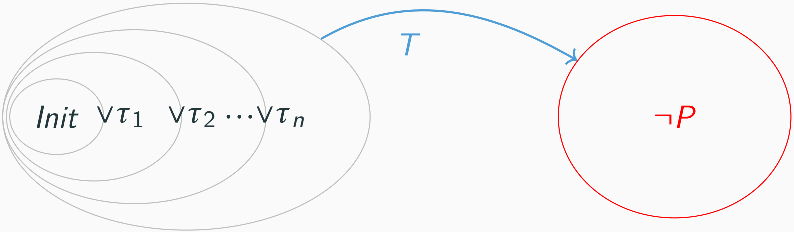

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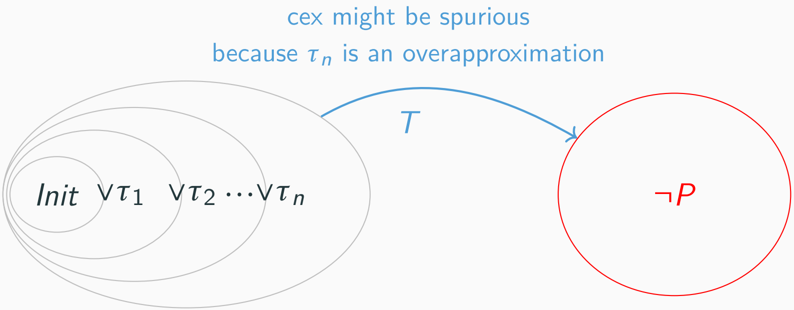
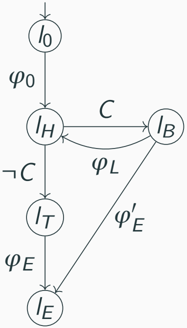


Termination Conditions of IMC

- IMC terminates when
  - A fixed point is reached: return a safety proof
  - $\mathit{Init}(s_0) \land \bigwedge_{i=1}^{k} T(s_{i-1}, s_i) \land (\bigvee_{i=1}^{k} \neg P(s_i))$ is SAT for some k: return a counterexample
- This work improves IMC's capability of constructing proofs

IMC for Software Verification

- Extract formulas by *large-block encoding* [5, 7]



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(For multi-loop programs: standard transformation to single loop [1, 12])

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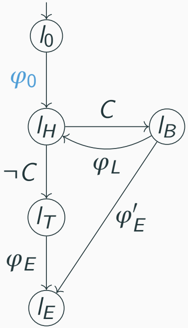

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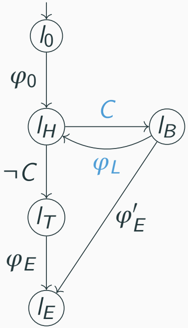

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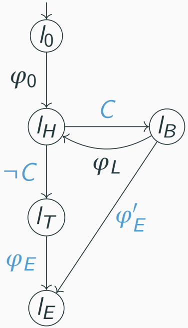

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Auxiliary Invariant Generator

- Continuously-refining data-flow analysis (DF) based on intervals [3]
- Produce **inductive** invariants
- Invariants are expressions over intervals
  - e.g. $(0 \le j \le 1) \land (x < 5 \lor x > 7)$
- Invariant injection denoted as $\leftarrow\!\circlearrowleft\!-$DF

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- τ'_i is a valid interpolant for IMC
- Inv helps remove some **unreachable** states in τ_i
- $\tau'(s_0) \land \bigwedge_{i=1}^{k} T(s_{i-1}, s_i) \land (\bigvee_{i=1}^{k} \neg P(s_i))$ is more likely to remain UNSAT

Alternative Ways to Utilize Auxililiary Invariants

- Injecting auxiliary invariants into
  - Fixed-point check: $\boldsymbol{Inv} \wedge \tau_n \Rightarrow Init \vee \bigvee_{i=1}^{n-1} \tau_i$
  - Safety property: $P' \leftarrow P \wedge \boldsymbol{Inv}$

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  - Fixed-point check: $\boldsymbol{Inv} \wedge \tau_n \Rightarrow Init \vee \bigvee_{i=1}^{n-1} \tau_i$
  - Safety property: $P' \leftarrow P \wedge \boldsymbol{Inv}$
- Not as effective as strengthening interpolants

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Evaluation

- We conducted experiments to answer the following research questions:
  - **RQ1**: Can auxiliary invariants help improve IMC?
  - **RQ2**: Is the augmented IMC competitive?

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- We conducted experiments to answer the following research questions:
  - **RQ1**: Can auxiliary invariants help improve IMC?
  - **RQ2**: Is the augmented IMC competitive?
- Reproduction package [4] available and reusable!

Implementation and Configurations in CPAchecker

- CPAchecker[2]: revision 42901 of branch *imc-with-invariants*
- Interpolants computed by MathSAT5 [10] (theory: QF_ABVFPUF)

[2] https://cpachecker.sosy-lab.org/

Implementation and Configurations in CPAchecker

- CPACHECKER[2]: revision 42901 of branch *imc-with-invariants*
- Interpolants computed by MATHSAT5 [10] (theory: QF_ABVFPUF)
- Compared SMT-based algorithms
  - IMC [7] vs. IMC←⟲-DF
  - KI←⟲-DF [6], predicate abstraction [14], IMPACT [17]

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Benchmark Tools and Tasks

- Compared software verifiers (from SV-COMP 2022 [2])
  - 2LS [8]: *k*-induction boosted by auxiliary invariants
  - SYMBIOTIC [18]: the overall winner of SV-COMP 2022

Benchmark Tools and Tasks

- Compared software verifiers (from SV-COMP 2022 [2])
  - 2LS [8]: *k*-induction boosted by auxiliary invariants
  - Symbiotic [18]: the overall winner of SV-COMP 2022
- Benchmark set: *ReachSafety* tasks of SV-COMP 2022 [2]
  - No property violation (i.e., safe)
  - Focus on single-loop programs
  - Eliminate easy ones solvable by CPAchecker's BMC within 900 s
  - 1623 after filtering
  - DF can produce non-trivial invariants on 870 tasks

Experimental Setup

- Environment
  - OS: Ubuntu 22.04 (64 bit)
  - Machine: 3.4 GHz CPU (8 cores) and 33 GB of RAM
- Each task is limited to
  - 4 CPU cores
  - 900 s of CPU time (max 150 s for DF)
  - 15 GB of RAM

  (reliable resource management by BenchExec[3])

[3] `https://github.com/sosy-lab/benchexec`

RQ1: Can auxiliary invariants help improve IMC?

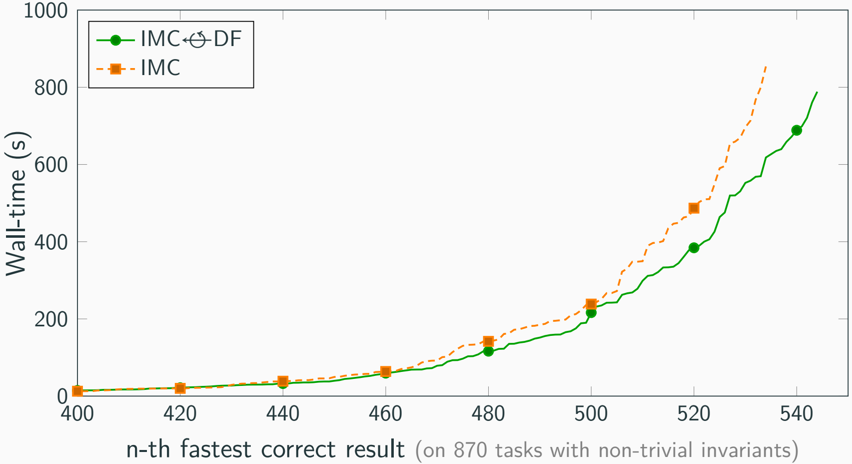
Improved Effectiveness

Task	IMC (timeout)			IMC⟲DF (solved)		
	#unroll	#itp	wall-time	#unroll	#itp	wall-time
Problem03_label51	10	57	878	5	11	24.3
benchmark37_conj	317	316	892	1	2	1.83
s3_srvr_1a.BV.c.cil	64	441	885	5	13	6.26

(time unit: s; hand-picked tasks with significant improvement)

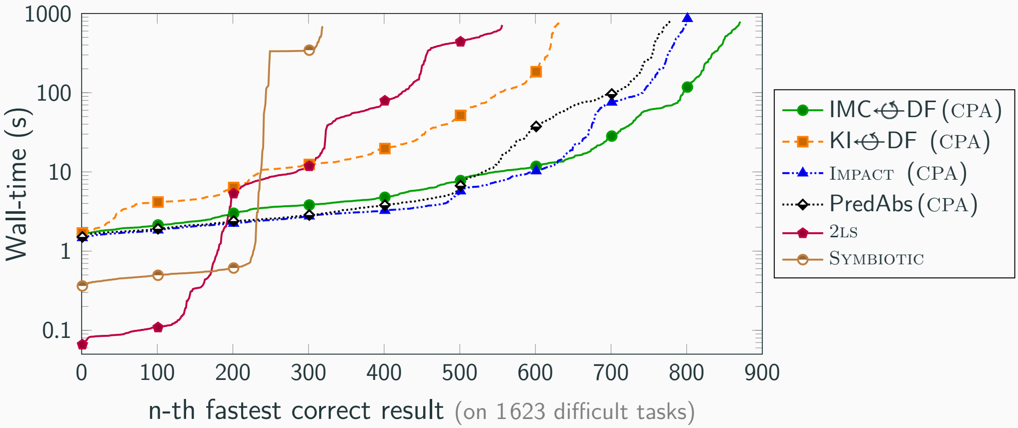
- IMC⟲DF solved **23** tasks where plain IMC ran into timeouts
  (on tasks with non-trivial auxiliary invariants)

Comparing Elapsed Wall-Time



RQ2: Is the augmented IMC competitive?

Comparsion with Others



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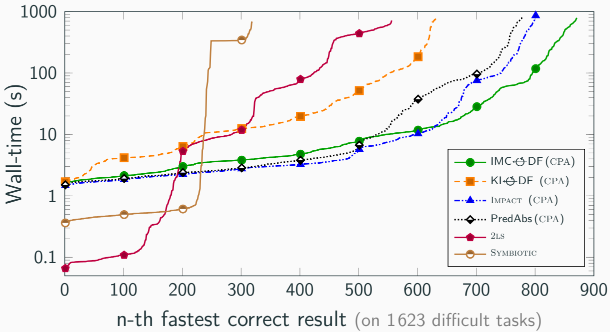
  Yes, effectiveness and wall-time efficiency are improved
- **RQ2**: Is the augmented IMC competitive?
  Yes, more proofs compared to other verification algorithms and tools

Conclusion

- Augment IMC [16] via invariant injection
- Open-source implementation in CPACHECKER



www.sosy-lab.org/research/imc-df/

References i

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Example Revisited: Collecting Formulas

```
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

- Large-block encoding [5]
- $s \leftarrow \{x, j\}$

Example Revisited: Collecting Formulas

I

```
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

- Large-block encoding [5]
- $s \leftarrow \{x, j\}$
- $Init(s) \leftarrow (x = 0) \land (j = 0)$

Example Revisited: Collecting Formulas

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

- Large-block encoding [5]
- $s \leftarrow \{x, j\}$
- $Init(s) \leftarrow (x = 0) \land (j = 0)$
- $T(s, s') \leftarrow (x_1 = x + 2)$
  $\land (j = 3 \Rightarrow x' = x_1 + 1)$
  $\land (j \neq 3 \Rightarrow x' = x_1)$
  $\land (j_1 = j + 1)$
  $\land (j_1 = 2 \Rightarrow j' = 0)$
  $\land (j_1 \neq 2 \Rightarrow j' = j_1)$

Example Revisited: Collecting Formulas

```
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

- Large-block encoding [5]
- $s \leftarrow \{x, j\}$
- $Init(s) \leftarrow (x = 0) \wedge (j = 0)$
- $T(s, s') \leftarrow (x_1 = x + 2)$
  $\wedge (j = 3 \Rightarrow x' = x_1 + 1)$
  $\wedge (j \neq 3 \Rightarrow x' = x_1)$
  $\wedge (j_1 = j + 1)$
  $\wedge (j_1 = 2 \Rightarrow j' = 0)$
  $\wedge (j_1 \neq 2 \Rightarrow j' = j_1)$
- $P(s') \leftarrow x' \% 2 = 0$

Example Revisited: Plain IMC

```c
int main(void) {
```

I

```c
  unsigned x = 0;
  unsigned j = 0;
```

T

```c
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
```

P

```c
  if (x % 2) {
    ERROR: return 1;
  }
```

```c
  return 0;
}
```

One loop unrolling ($k = 1$)

- $Init \wedge T \wedge \neg P$ is UNSAT
- $\tau_1 \leftarrow x\%2 = 0$

Example Revisited: Plain IMC

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

(Brackets beside the code: I = lines 2–3, T = lines 4–11, P = lines 12–14)

One loop unrolling ($k = 1$)

- $Init \wedge T \wedge \neg P$ is UNSAT
- $\tau_1 \leftarrow x\%2 = 0$
- $\tau_1 \wedge T \wedge \neg P$ is SAT
  (spurious cex: $x = 0 \wedge j = 3$)

Example Revisited: Plain IMC

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

One loop unrolling ($k = 1$)

- $Init \land T \land \neg P$ is UNSAT
- $\tau_1 \leftarrow x \% 2 = 0$
- $\tau_1 \land T \land \neg P$ is SAT
  (spurious cex: $x = 0 \land j = 3$)
- Increment k

Example Revisited: Augmented IMC

```
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

One loop unrolling $(k = 1)$

- $Inv \leftarrow 0 \leq j \leq 1$

Example Revisited: Augmented IMC

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

One loop unrolling $(k = 1)$

- $Inv \leftarrow 0 \leq j \leq 1$
- $Init \wedge T \wedge \neg P$ is UNSAT
- $\tau_1 \leftarrow x\%2 = 0; \quad \boldsymbol{\tau'_1 \leftarrow \tau_1 \wedge Inv}$
- $\tau'_1 \wedge T \wedge \neg P$ is UNSAT

Example Revisited: Augmented IMC

```
      int main(void) {
I       unsigned x = 0;
        unsigned j = 0;
        while (nondet()) {
          x += 2;
          if (j == 3)
T           x += 1;
          j += 1;
          if (j == 2)
            j = 0;
        }
        if (x % 2) {
P         ERROR: return 1;
        }
        return 0;
      }
```

One loop unrolling $(k = 1)$

- $Inv \leftarrow 0 \leq j \leq 1$
- $Init \wedge T \wedge \neg P$ is UNSAT
- $\tau_1 \leftarrow x\%2 = 0$; $\boldsymbol{\tau'_1 \leftarrow \tau_1 \wedge Inv}$
- $\tau'_1 \wedge T \wedge \neg P$ is UNSAT
- $\tau_2 \leftarrow x\%2 = 0$; $\boldsymbol{\tau'_2 \leftarrow \tau_2 \wedge Inv}$

Example Revisited: Augmented IMC

```c
int main(void) {
  unsigned x = 0;
  unsigned j = 0;
  while (nondet()) {
    x += 2;
    if (j == 3)
      x += 1;
    j += 1;
    if (j == 2)
      j = 0;
  }
  if (x % 2) {
    ERROR: return 1;
  }
  return 0;
}
```

One loop unrolling $(k = 1)$

- $Inv \leftarrow 0 \leq j \leq 1$
- $Init \wedge T \wedge \neg P$ is UNSAT
- $\tau_1 \leftarrow x\%2 = 0; \quad \boldsymbol{\tau'_1 \leftarrow \tau_1 \wedge Inv}$
- $\tau'_1 \wedge T \wedge \neg P$ is UNSAT
- $\tau_2 \leftarrow x\%2 = 0; \quad \boldsymbol{\tau'_2 \leftarrow \tau_2 \wedge Inv}$
- $\tau'_2 \Rightarrow (I \vee \tau'_1)$ holds: fixed point!

Critical Reflection

- Auxiliary invariants brought *net improvement* to IMC
- However...

Critical Reflection

- Auxiliary invariants brought *net improvement* to IMC
- However...
  - The improvement was not remarkable
  - Some tasks became unsolvable with added invariants
- Reasons:
  - Invariant generator consumed additional CPU time
  - Interpolation queries became more difficult for the solver