



Domain-Independent Interprocedural Program Analysis using Block-Abstraction Memoization

Dirk Beyer
LMU Munich, Germany

Karlheinz Friedberger
LMU Munich, Germany

ABSTRACT

Whenever a new software-verification technique is developed, additional effort is necessary to extend the new program analysis to an interprocedural one, such that it supports recursive procedures. We would like to reduce that additional effort. Our contribution is an approach to extend an existing analysis in a modular and domain-independent way to an interprocedural analysis without large changes: We present *interprocedural* block-abstraction memoization (BAM), which is a technique for procedure summarization to analyze (recursive) procedures. For recursive programs, a fix-point algorithm terminates the recursion if every procedure is sufficiently unrolled and summarized to cover the abstract state space.

BAM Interprocedural works for data-flow analysis and for model checking, and is independent from the underlying abstract domain. To witness that our interprocedural analysis is generic and configurable, we defined and evaluated the approach for three completely different abstract domains: predicate abstraction, explicit values, and intervals. The interprocedural BAM-based analysis is implemented in the open-source verification framework CPACHECKER. The evaluation shows that the overhead for modularity and domain-independence is not prohibitively large and the analysis is still competitive with other state-of-the-art software-verification tools.

CCS CONCEPTS

• **Software and its engineering** → **Formal methods; Formal software verification**; • **Theory of computation** → **Program verification; Verification by model checking**.

KEYWORDS

Software Verification, Interprocedural Program Analysis, Recursive C Program, Block Abstraction, Procedure Summary

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1 INTRODUCTION

Software verification has been successfully applied to improve the quality and reliability of computer programs [2, 3, 19, 22, 28, 30, 40]. In the last decades, several algorithms and approaches were developed to perform software model checking for various kinds of C programs. However, only a few verifiers for C support full interprocedural analysis, that is, verification of recursive programs: Only 13 out of 22 tool submissions (17 different tools) in the 2020 competition on software verification [5] participated successfully in the benchmark category of recursive tasks.

A program analysis is called *interprocedural* if procedures are analyzed separately and verification results are merged together from the separate results. The idea is that a program analysis does not depend on long traces through the program, but analyzes procedures independently from each other, such that the result of a procedure's analysis can be used at all call sites with the same context (e. g., with the same abstract arguments). Many verifiers inline called procedures into the calling procedure and verify long traces through a program without any benefit from a modular approach. This not only hinders the reuse of sub-results of the analysis, but also makes it impossible to verify unbounded recursive programs.

We present BAM Interprocedural, a generalization of summary-based interprocedural analysis. The abstract framework is an extension of block-abstraction memoization (BAM) [9, 57] and is currently used to verify reachability properties about programs.

Example. We outline how to prove the correctness of the example program in Fig. 1 (illustrated in Fig. 2), which uses two unsigned integer variables a and b , and nondeterministically initializes them as input for the recursive procedure *sum*, which returns the sum of its arguments. The program is deemed correct if *error()* is not called.

This program can not be verified by a default bounded model checker that iteratively unrolls the recursion, because the number of unrollings is unknown. However, using a procedure summary like $ret = m + n$, where m and n are the parameters of procedure *sum* and *ret* is the return value of the procedure call, would help with the verification. This summary is a valid abstraction for the control-flow for every call of the procedure *sum* and can be applied as a substitution for the initial call in procedure *main* as well as for the recursive call in procedure *sum* itself. For a fully automated analysis, the verification algorithm must come up with this (or some similar) summary and apply it as part of the proof strategy.

This example program requires an abstract domain that tracks relations between variables. Thus, a standard predicate analysis (such as in Sect. 4) is able to infer such predicates (e. g., via CEGAR [27] and interpolation [43]) and can soundly apply procedure summaries for all call-sites of a procedure. In general, our approach works on a domain-independent level and does not depend on SMT-based summaries. The combination of procedure summaries with a fixed-point algorithm computes an over-approximation of the reachable

```

1 void main(void) {
2   uint a = nondet();
3   uint b = nondet();
4   uint s = sum(a, b);
5   if (s != a + b) {
6     error();
7   }
8 }
9
10 uint sum(uint n, uint m) {
11   if (n == 0) {
12     return m;
13   } else {
14     uint tmp = sum(n - 1, m + 1);
15     return tmp;
16   }
17 }

```

Figure 1: Example program with a recursive procedure *sum*

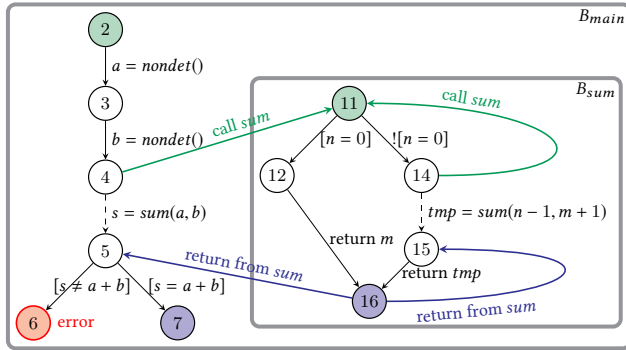


Figure 2: CFAs for the example program in Fig. 1, with procedure blocks B_{main} and B_{sum}

state space of the recursive procedure. The algorithm first determines a procedure summary for a single unrolling of the procedure, i. e., for all paths through the procedure that are not traversing the recursive call. Using the above mentioned abstract domain, the analysis obtains a summary like $ret = m + n$ in this first step. Then, the algorithm applies the computed procedure summary to the recursive call and explores longer paths through the program and refines the procedure summary until the algorithm cannot explore any new path. For the given example, applying this summary once for the recursive procedure call within the procedure *sum* does not change the summary of the whole procedure *sum*, thus it is sufficient to reach a fixed-point and the analysis can terminate.

Contribution. Our contribution consists of three parts:

(1) We present a *domain-independent* approach of BAM [57] for a fully interprocedural analysis: every procedure is analyzed separately and the result of a procedure’s analysis (an abstraction of the procedure, also known as “procedure summary”) is integrated in the analysis of the calling context.

(2) A program might contain *unbounded recursion* (e. g., the recursion depth is depending on unknown input). Instead of just cutting off program traces at a predefined depth, our analysis terminates the unrolling of a recursive procedure in a sound way once

a fixed point is reached, and does not omit feasible error paths. The fixed-point algorithm iteratively increments the unrolling of the recursion until no new abstract state is reachable. The algorithm is domain-independent, because only coverage checks for abstract states are used, which are already provided by each abstract domain. The overhead is negligible for non-recursive programs.

(3) We formally define an *additional domain-specific operator* rebuild in the framework, such that recursive procedures can be handled in every domain. This operator restores eliminated information of the calling context after leaving a recursive call.

Related Work. As programs with (recursive) procedures have been analyzed and also verified since decades, many ideas are already available and implemented in some tools. We give a short overview of the tools and the domains they are based on.

Inlining-Based Analysis. A common approach to analyze procedures in bounded model checking is to unroll them up to a certain limit and ignore any deeper recursive calls. Tools like CBMC [29], ESbMC [36], and SMACK [47] implement this approach, which leads to an unsound analysis in combination with recursive procedure calls, because there is no guarantee that the bug is unreachable through further unrolling. Without the user specifying a bound, the model checker might run into an endless unrolling of the recursion. Constant propagation (like in CBMC) or additional checks can avoid too far unrolling of recursive procedures. Also unbounded frameworks like CPACHECKER [15] have several analyses based on different domains [16, 17, 46] that inline procedure calls. Our approach is built on top of them and reuses existing components, such that the amount of changes to a single analysis is minimal.

Interpolation-Based Summaries. Some approaches to verify recursive programs start with the analysis of single procedures and compute procedure summaries when applying nested function calls. The bounded model checker FUNFROG [53, 54] generates interpolation-based [33] procedure summaries to avoid the repeated analysis of procedures. WHALE [1] is an extension of IMPACT [44] and analyzes recursive procedures using two types of formulas in its intra-procedural analysis, namely state- and transition-interpolants, to get summaries. Those approaches separately analyze each procedure until a fixed-point is reached and the procedures (or the representing formulas) are sufficiently refined. UAUTOMIZER uses nested interpolants [38] to compute formulas for procedures depending on the caller’s context. Those approaches are bound to an SMT-based domain and the algorithms do not support combinations with other domains.

Further Domain-Specific Interprocedural Analyses. BEBOP [4] computes procedure summaries for boolean programs. The application of BEBOP however is limited to boolean programs and abstract states are described with binary decision diagrams. ABDUCTOR [23] is an interprocedural program verifier that applies the domain of separation logic to prove memory-related safety properties. Additionally, a recursive program can be transformed into a non-recursive one, such that any verification tool without direct support for recursion can be used indirectly to analyze the recursive program. For example, CPAREC [26] is a light-weight approach using an external black-box verifier and a fixed-point algorithm that increments the unrolling depth to compute procedure summaries until coverage is reached. This approach is limited to predicate-based verifiers.

Interprocedural Data-Flow Analysis. The above examples are based on symbolic analysis, i. e., depending on BDD-, SAT-, or SMT-based domains, while our proposed approach works for classic data-flow domains as well. Since many years, programs were analyzed in interprocedural manner using several lattice-based domains [31, 50] and with procedure summaries [55]. The classic approach to interprocedural data-flow analysis [21, 41, 48] is restricted to finite-height lattices of domain elements and an operator yielding the join of two domain elements.

BAM Interprocedural works for arbitrary, unlimited abstract domains and different operators for combining elements (depending on the represented data, not only join) or coverage checks for elements (domain-specific comparison).

2 BACKGROUND

We describe the program representation as a control-flow automaton and domain-independent reachability analysis based on the concept of configurable program analysis. Afterwards, their application as components in an interprocedural analysis is shown.

2.1 Programs

A *program* is represented by a *control-flow automata* (CFA) $A = (L, l_0, G)$ that consists of a set L of program locations, an initial program location $l_0 \in L$, and a set $G \subseteq L \times Ops \times L$ of control-flow edges. An edge models the control-flow operation (from *Ops*) between program locations, for example assignments or assumptions. Figure 2 represents the example program as CFAs. Our presentation uses a simple imperative programming language, which allows only assignments, assume operations, procedure calls and returns, and all variables are integers. The implementation of our tool provides basic support for heap-related data-structures including pointers and arrays, but this article avoids them for simplicity. In general, CPACHECKER [15] supports the verification of C programs including pointers and arrays. However, the analysis of recursive procedures for such programs is still under development and a topic of research.

2.2 Blocks in a Program

Blocks are formally defined as parts of a program: A block $B = (L', G')$ of a CFA $A = (L, l_0, G)$ consists of a set $L' \subseteq L$ of program locations and a set $G' = \{(l_1, op, l_2) \in G \mid l_1, l_2 \in L'\}$ of control-flow edges. We assume that two blocks B and B' are either disjoint ($B.L' \cap B'.L' = \emptyset$) or one block is completely nested in the other block ($B.L' \subset B'.L'$). Each block has *input* and *output locations*, which are defined as $In(B) = \{l \in L' \mid (\exists l': (l', \cdot, l) \in G \wedge l' \notin L') \vee (\exists l': (l', \cdot, l) \in G)\}$ and $Out(B) = \{l \in L' \mid (\exists l': (l, \cdot, l') \in G \wedge l' \notin L') \vee (\exists l': (l, \cdot, l') \in G)\}$, respectively. In general, the block size can be freely chosen in our approach. For an interprocedural analysis, we use procedures as blocks, such that a block abstraction represents a procedure summary. In Fig. 2, the blocks B_{main} and B_{sum} represent the two procedures of the program. The input and output locations are marked in color for each block.

2.3 CPA and CPA Algorithm

The reachability analysis is based on the concept of configurable program analysis (CPA) [13], which specifies the abstract domain for a program analysis and additional operations.

A CPA $\mathbb{D} = (D, \rightsquigarrow, \text{merge}, \text{stop})$ consists of an abstract domain D , a transfer relation \rightsquigarrow , and the operators *merge* and *stop*. The abstract domain $D = (C, \mathcal{E}, \llbracket \cdot \rrbracket)$ consists of a set C of concrete states, a semi-lattice $\mathcal{E} = (E, \sqsubseteq)$ over a set E of abstract-domain elements (i. e., abstract states) and a partial order \sqsubseteq (the join \sqcup of two elements and the join \top of all elements are unique), and a concretization function $\llbracket \cdot \rrbracket : E \rightarrow 2^C$ that maps each abstract-domain element to the represented set of concrete states. The transfer relation $\rightsquigarrow \subseteq E \times E$ computes abstract successor states, a transfer relation $\overset{g}{\rightsquigarrow}$ matches the transfer along an edge $g \in G$ of the CFA. The merge operator $\text{merge} : E \times E \rightarrow E$ specifies if and how to merge two abstract states when control flow meets. The stop operator $\text{stop} : E \times 2^E \rightarrow \mathbb{B}$ determines whether an abstract state is covered by a given set of abstract states. The operators *merge* and *stop* can be chosen appropriately to influence the abstraction level of the analysis. Common choices include $\text{merge}^{sep}(e, e') = e'$ (which does not merge abstract states) and $\text{stop}^{sep}(e, R) = (\exists e' \in R : e \sqsubseteq e')$ (which determines coverage by checking whether the given abstract state is less than or equal to any other reachable abstract state according to the semi-lattice).

Given a CPA, we can apply a reachability algorithm (denoted as CPA algorithm in [13]) that explores the abstract state space of a program and computes all reachable abstract states. The stop operator determines the fixed-point criteria, i. e., whether a state has already been discovered before. For the following description, we consider a reachability analysis $CPA(\mathbb{D}, \text{reached}, \text{waitlist})$ using a CPA \mathbb{D} and two sets *reached* and *waitlist* of abstract states as input and returning two sets *reached'* and *waitlist'* of abstract states. The idea is that starting with the given sets of already reached abstract states and a frontier *waitlist*, the reachability algorithm computes more reachable successors and a new frontier *waitlist*.

The CPA algorithm can be used as component in a CEGAR-based fixed-point loop [27] to refine the granularity of the current analysis. For simplicity we ignore the precision in this article.

In the following Sect. 3, we describe our interprocedural extension of block-abstraction memoization, and then in Sect. 4 provide an application of the concept to three separate domains: the Callstack-CPA for tracking a call stack of the program, the Value-CPA for tracking variable assignments explicitly, and the Predicate-CPA for handling variable assignments with predicates.

3 BAM FOR INTERPROCEDURAL ANALYSIS

Block-Abstraction Memoization (BAM) [57] is a modular and scalable approach for model checking abstract state spaces by leveraging the idea of *divide and conquer*. BAM divides a large program into smaller parts, named *blocks*, and analyzes them separately. The result of a block's analysis is denoted as a *block abstraction*. Block abstractions are stored in a cache. Whenever a larger block depends on a nested block, a block abstraction of the nested block is created during the larger block's analysis. Block abstractions are independent of a concrete domain and work on an abstract level.

There can be several block abstractions for the same block, e. g., depending on different input values of the block.

In the following, we use procedures as blocks. More precisely, a *procedure block* B_f consists of the procedure f itself and all procedures that are (transitively) called from f , such that the whole control-flow of nested blocks, including call and return edges, is included in the block B_f (see Fig. 2).

BAM ensures efficiency by using a cache $\text{cache} \subseteq (\text{Blocks} \times E) \rightarrow (2^E \times 2^E)$ for block abstractions, which maps the initial abstract state for a block to the block abstraction. The block abstraction is defined as the set of reached abstract states and the set of frontier abstract states, which both are computed during the block's analysis.

BAM is defined recursively (independent of any recursion in the analyzed program) and repeatedly (nestedly) applies the reachability analysis. Our implementation of BAM uses a *stack* of pairs $p \in \text{Blocks} \times E$ that consists of all currently open analyses referenced by their block of the CFA to be analyzed and an initial abstract state (starting point of the block abstraction).

This section defines BAM Interprocedural. We show that procedure blocks correspond to procedure summaries, describe the problems of analyzing recursive procedures, the necessity of the fixed-point algorithm, and a new operator rebuild.

3.1 Operators of BAM

BAM uses two complementing operators $\text{reduce} \subseteq \text{Blocks} \times E \rightarrow E$ and $\text{expand} \subseteq \text{Blocks} \times E \times E \rightarrow E$, and an additional operator $\text{rebuild} \subseteq E \times E \times E \rightarrow E$, to drop or restore context-based information for each analyzed block. A CPA with these three additional operators is called *CPA with BAM operators*. On an abstract level, the reduce operator performs an abstraction of the given abstract state and the expand operator concretizes an abstract state for a given context. These operators aim towards an interprocedural analysis where each block can be analyzed without knowing its concrete context. How much of this context-independence can be achieved depends on the concrete domain (see Sect. 4 for more details). The implicit benefit of the first two operators is an improvement of the cache-hit-rate. The operator reduce drops unimportant information from an abstract state when entering a block. The resulting abstract state is more abstract and is used as cache key and as initial abstract state for the block's analysis. The importance of some information depends on the wrapped analysis and the available block. For example, variables, predicates, or levels of the call stack that are not accessed inside the entered block, but only depend on the surrounding context, might be good candidates to be removed from the abstract state. The operator expand restores removed information for abstract states when applying the block abstraction in the surrounding context. The operator rebuild avoids collisions of program identifiers (like variables) when returning from a (possibly recursive) procedure scope into its calling context. This operator does not compute an abstraction, but performs simple operations depending on the given abstract domain such as renaming variables, substituting predicates, or updating indices.

With these operators, we now formally define the CPA for BAM.

Algorithm 1 $\text{fixedPoint}(B_{\text{main}}, l_0, e_0)$

Input: block B_{main} with initial program location l_0 , abstract state e_0
Output: set of reachable states, which all represent output states of the block B_{main}

Global Variables: boolean flag fixedpointReached

Variables: set blockResult of abstract states

```

1: repeat
2:    $\text{fixedpointReached} := \text{true}$ ;
3:    $\text{blockResult} := \text{applyBlockAbstraction}(B_{\text{main}}, e_0)$ ;
4: until  $\text{fixedpointReached}$ 
5: return  $\text{blockResult}$ ;
```

3.2 BAM as CPA

For usage with the CPA concept (see Sect. 2.3), BAM itself is formalized as a CPA $\text{BAM} = (D_{\text{BAM}}, \rightsquigarrow_{\text{BAM}}, \text{merge}_{\text{BAM}}, \text{stop}_{\text{BAM}})$. As BAM works on an abstract, domain-independent level, it requires a separate abstract-domain-dependent analysis (like the *value analysis* or *predicate analysis*) to track variables, values, and assignments. This separate component analysis is also defined via the CPA concept (see Sect. 4). For the following definition we denote it as a general (wrapped) CPA with BAM operators $\mathbb{W} = (D_{\mathbb{W}}, \rightsquigarrow_{\mathbb{W}}, \text{merge}_{\mathbb{W}}, \text{stop}_{\mathbb{W}}, \text{reduce}_{\mathbb{W}}, \text{expand}_{\mathbb{W}}, \text{rebuild}_{\mathbb{W}})$.

- (1) The domain D_{BAM} is the wrapped domain $D_{\mathbb{W}}$, i. e., BAM simply uses the abstract states of the underlying domain.
- (2) The transfer relation includes the transfer $e \rightsquigarrow_{\text{BAM}} e'$ for two abstract states e and e' and a block B if

$$e' \in \begin{cases} \text{fixedPoint}(B_{\text{main}}, l, e) & \text{if } l = l_0 \text{ and } \text{stack} = [] \\ \text{applyBlockAbstraction}(B, e) & \text{if } l \in \text{In}(B) \\ \{e'' \mid e \rightsquigarrow_{\mathbb{W}} e''\} & \text{if } l \notin \text{Out}(B) \end{cases}$$

where l is the program location for e and stack is the internal stack of nested blocks during the analysis.

The transfer relation applies one of three possible steps:

- (1) The fixed-point algorithm Alg. 1 is executed if the current program location is the initial program location l_0 and the stack is empty.
- (2) At an input location of a block B , i. e., if a new nested block would be entered from a surrounding context, we apply the block abstraction returned from the operation $\text{applyBlockAbstraction}$ (cf. Alg. 2) for the nested block.
- (3) For output locations of blocks, there is no succeeding abstract state (in the sub-analysis). For other program locations, the wrapped transfer relation $\rightsquigarrow_{\mathbb{W}}$ is applied.
- (3) The merge operator $\text{merge}_{\text{BAM}} = \text{merge}_{\mathbb{W}}$ delegates to the wrapped analysis, i. e., BAM merges whenever the underlying domain merges abstract states.
- (4) The termination check $\text{stop}_{\text{BAM}} = \text{stop}_{\mathbb{W}}$ delegates to the wrapped analysis, i. e., the coverage relation between abstract states depends on the underlying domain.

The transfer relation $\rightsquigarrow_{\text{BAM}}$ uses the fixed-point algorithm and the computation of block abstractions as explained in the next subsections.

Algorithm 2 *applyBlockAbstraction*(B, e_I)

Input: abstract state e_I at a block input location of a block B
Output: abstract states for the output locations of the analyzed block B
Global Variables: boolean flag `fixedpointReached`,
 set cache mapping a block and an abstract state to a
 block abstraction,
 sequence stack consisting of pairs of a procedure block
 and an abstract state

Variables: sets `reached` and `waitlist` of abstract states
 for the analysis of the current block

- 1: $e_i := \text{reduce}_{\mathbb{W}}(B, e_I)$;
- 2: **if** $\exists(B, e_c) \in \text{stack} : e_i \sqsubseteq e_c$ **then**
- 3: **if** cache contains (B, e_c) **then**
- 4: $(\text{reached}, \cdot) := \text{cache}(B, e_c)$;
- 5: **else**
- 6: $\text{reached} := \{\}$
- 7: $\text{fixedpointReached} := \text{false}$;
- 8: **else**
- 9: **if** cache contains (B, e_i) **then**
- 10: $(\text{reached}, \text{waitlist}) := \text{cache}(B, e_i)$
- 11: **else**
- 12: $\text{reached} := \{e_i\}$; $\text{waitlist}_r := \{e_i\}$
- 13: $\text{stack.push}(B, e_i)$;
- 14: $(\text{reached}, \text{waitlist}) := \text{CPA}(\mathbb{W}, \text{reached}, \text{waitlist})$
- 15: $\text{stack.pop}()$;
- 16: **if** cache contains (B, e_i) **then**
- 17: $(\text{reached}_{old}, \cdot) := \text{cache}(B, e_i)$;
- 18: **for** $e \in \text{reached}$ **do**
- 19: **if** $\text{loc}(e) \in \text{Out}(B) \wedge \nexists e' \in \text{reached}_{old} : e \sqsubseteq e'$ **then**
- 20: $\text{fixedpointReached} := \text{false}$;
- 21: $\text{cache}(B, e_i) := (\text{reached}, \text{waitlist})$
- 22: $e_{call} := \text{getPredecessor}(e_I)$;
- 23: $\text{tmp} := \{\text{expand}_{\mathbb{W}}(B, e_I, e_o) \mid e_o \in \text{reached} \wedge \text{loc}(e_o) \in \text{Out}(B)\}$
- 24: **return** $\{\text{rebuild}_{\mathbb{W}}(e_{call}, e_I, e_O) \mid e_O \in \text{tmp}\}$;

3.3 Fixed-Point Algorithm for Unbounded Recursion

An analysis of recursive procedures must handle a possibly unbounded unrolling of the call stack if the information of an abstract state is insufficient to avoid deeper exploration and can not cut off the state space. In our approach, the fixed-point algorithm (*fixedPoint*, Alg. 1) repeatedly analyzes the program using *applyBlockAbstraction* (Alg. 2) from the initial program location onwards. It iteratively increments the number of unrollings and terminates only if coverage was reached for all analyzed procedure calls.

In each iteration of the fixed-point algorithm, we generate an overapproximation of some (more) paths through the recursive procedure (because of the limited unrolling of the recursion) and determine a summary for the currently analyzed procedure block. The termination is decided by a coverage check for the abstract states of the analyzed block summary.

The first iteration of the fixed-point algorithm assumes no valid path through the recursive call. We only explore the non-recursive parts of the program's control flow and skip the recursive call of the procedure. Depending on the abstract domain, the initial summary for the recursive procedure is an empty set of abstract states (Alg. 2, line 6). The block abstraction of a procedure is stored in the cache after returning from the procedure call (Alg. 2, line 21).

$$\frac{\{P\}b = f(a)\{Q\} \quad \vdash \quad \{P \wedge p = a\}B_f\{Q \wedge p = a \wedge b = r\}}{\{P\}b = f(a)\{Q\}}$$

Figure 3: Hoare's rule for recursion, for a given procedure definition $f(p) \{B_f; \text{return } r;\}$

$$\frac{\{\llbracket P_e \rrbracket\}b = f(a)\{\llbracket Q_e \rrbracket\} \quad \vdash \quad \{\llbracket P_e \rrbracket\}B_f\{\llbracket Q_e \rrbracket\}}{\{\llbracket P_e \rrbracket\}b = f(a)\{\llbracket Q_e \rrbracket\}}$$

Figure 4: Hoare's rule for recursion (with abstract states)

In further iterations, we increment the limit of unrollings of the recursive procedure and refine the block abstraction, analyze the program again, starting from the initial program location (and using several intermediate results from the cache), until the procedure summary becomes stable.

3.4 Soundness of BAM for Recursion

The fixed-point criteria are based on Hoare's rule for recursion (Fig. 3): if the body of a procedure f satisfies the pre- and post-conditions P and Q (including parameter passing and return values) under the condition that all recursive calls to the procedure f satisfy P and Q , then the whole procedure f satisfies P and Q . Translated into our model, we use (concretizations of) abstract states as pre- and post-conditions of statements, the procedure and its body corresponds to the procedure's block; Fig. 4 shows the resulting rule. The renaming (or an equivalent operation) of equal identifiers from the recursive call of f , which appear in the calling and called procedure f , is shifted into a different part of the analysis (see Sect. 3.5 on operator rebuild) and is handled in a sound way.

To determine the fixed-point criteria for termination, Alg. 2 checks the following two properties during the analysis.

Firstly, we try to stop the unrolling of an unbounded recursive procedure by an over-approximating analysis. Thus, before analyzing a new recursive procedure call, we check whether the abstract state at a procedure entry is already covered by any abstract state from the current stack (Alg. 2, line 2). If such a covering abstract state exists, we skip the recursive call and use a procedure summary instead of further exploring the recursive call (Alg. 2, line 3 to 7). The procedure summary consists of either previously computed abstract successor states from the BAM cache or (in case of a cache miss) no successor states at all.

Secondly, because a procedure summary represents only a bounded execution of the called procedure, this approach alone represents only a subset of possible traces in the procedure and might be unsound in cases that require deeper unrolling. To determine if the inserted procedure summaries are sufficient for Hoare's rule of Fig. 4, we check for coverage of the exit state (of the procedure executed with the inserted procedure summary) against the previously computed abstract states (of the procedure summary). This check is performed in lines 18 to 20 of Alg. 2. If the coverage relation is satisfied (for all procedures in the program), then the fixed-point algorithm terminates, because `fixedpointReached` was never set to `false` during the iteration. In this case we have found a sound over-approximation of the recursive procedure. Otherwise the fixed-point algorithm continues.

3.5 Block-Abstraction Computation with Operators

The operation *applyBlockAbstraction* (cf. Alg. 2) starts with the reduction $\text{reduce}_{\mathbb{W}}(B, e_I)$ of initial abstract state e_I and determines the block abstraction for a block B . The block abstraction is either taken from the cache or computed via a separate application of the reachability algorithm (i. e., CPA algorithm). To integrate the block abstraction into a surrounding context, the operators $\text{expand}_{\mathbb{W}}$ and $\text{rebuild}_{\mathbb{W}}$ are applied to each abstract state at the block's output location (lines 23 and 24). The operators reduce and expand abstract or concretize the given abstract state and aim to increase the cache-hit rate of BAM. For an interprocedural approach, they remove and restore (most of) the context-based information of a procedure block.

While the fixed-point algorithm handles over-approximations and refinements of block abstractions, an interesting detail of the implementation remains open: How can we identify and work with symbols, i. e., variable identifiers, across procedure scopes? Identical identifiers for program variables of the same procedure scope are problematic for the analysis of recursive procedures. Due to the modularity of the framework CPACHECKER, only a separate call-stack analysis knows about procedure scopes and all other analyses assume unique identifiers across all operations. BAM also tracks information about procedures in its stack, but it does not use this information for detailed analysis of variables and identifiers. Each recursive procedure entry starts a new procedure scope, where the identifiers override existing (valid) identifiers from previous call-stack levels. Entering a procedure and overriding existing identifiers from the calling scope is no problem, because only the most local version of an identifier is available (and visible) in the procedure scope. Leaving the procedure afterwards is more complex, because identifiers are overridden during the procedure's traversal and have to be restored to match the calling context.

A solution like a simple renaming of identifiers is not possible, because each domain has its own way of representing variables. Additionally, each domain must have a strategy for handling scoped variables that allows a consistent use of the cache in BAM.

We solve this problem by using a new operator $\text{rebuild} : E \times E \times E \rightarrow E$, and we show how to implement it for different domains. The operator rebuild is applied after analyzing the procedure-exit location (Alg. 2, line 24), i. e., after leaving the block of a (maybe recursive) procedure and after the application of the operator expand. The operator rebuild maps three abstract states (information about the calling context from the procedure call state e_{call} , information about the arguments and parameters of the called procedure from the procedure entry state e_I , and information about the return value and the block abstraction from the procedure exit state e_O) to a new abstract state that is a successor of the procedure call and a valid starting point for the further analysis. The operator rebuild is defined depending on the underlying analysis.

4 APPLICATION OF BAM INTERPROCEDURAL TO ABSTRACT DOMAINS

In this section, we describe some component program analyses that can be used by BAM Interprocedural to compute context-independent block abstractions. Using the framework CPACHECKER,

program analyses are composed of several component CPAs. Component CPAs are defined and implemented for tracking the program counter, the predecessor-successor relationship of the reachability graph, or for combining other CPAs in a composite analysis. Thus, we do not need to specify these aspects when defining a component analysis, but directly specify the component analyses. In the following, we explain an analysis for tracking the call stack and two analyses for analyzing variables and assignments (namely *value analysis* and *predicate analysis*).

Callstack-CPA. The CPA with BAM operators for call-stack analysis $\mathbb{C} = (D_{\mathbb{C}}, \sim_{\mathbb{C}}, \text{merge}_{\mathbb{C}}, \text{stop}_{\mathbb{C}}, \text{reduce}_{\mathbb{C}}, \text{expand}_{\mathbb{C}}, \text{rebuild}_{\mathbb{C}})$ explicitly tracks the call stack $s = [f_1, \dots, f_n]$ of the program, where f_1 to f_n denote procedure scopes for an abstract state s .

- (1) The domain $D_{\mathbb{C}} = (C, \mathcal{E}_{\mathbb{C}}, \llbracket \cdot \rrbracket)$ is based on the flat semi-lattice $\mathcal{E}_{\mathbb{C}} = (S \cup \{\top\}, \sqsubseteq)$ for the set S of possible call stacks. The expression $s \sqsubseteq s'$ is fulfilled if $s = s'$ or $s' = \top$, $\llbracket \top \rrbracket = C$. For all s in S , we have $\llbracket s \rrbracket = \{c \in C \mid \text{callstackOf}(c) = s\}$.
- (2) The transfer relation $\sim_{\mathbb{C}}$ has the transfer $s \xrightarrow{g}_{\mathbb{C}} s'$ for CFA edge g and abstract states $s = [f_1, \dots, f_{n-1}, f_n]$ and s' , if

$$s' = \begin{cases} [f_1, \dots, f_n, f_{n+1}] & \text{if } g \text{ is a procedure call to } f_{n+1} \\ [f_1, \dots, f_{n-1}] & \text{if } g \text{ is a procedure return from } f_n \\ s & \text{otherwise} \end{cases}$$
- (3) The merge operator $\text{merge}_{\mathbb{C}} = \text{merge}^{sep}$ does not combine abstract states.
- (4) The termination check $\text{stop}_{\mathbb{C}} = \text{stop}^{sep}$ returns whether the same abstract state was already reached before.
- (5) The reduce operator $\text{reduce}_{\mathbb{C}}$ abstracts from a concrete call stack and keeps only the context-relevant suffix. Therefore, it determines the maximal range of procedure scopes of the current block, i. e., procedure scopes that can be popped from the current call stack $[f_1, \dots, f_i, \dots, f_n]$ during an analysis of the current block. Let the procedure scope f_i be the lowest procedure scope on the stack that is reachable during the block's analysis. Then, the operator keeps only the reachable (most local) procedure scopes from the abstract state: $\text{reduce}_{\mathbb{C}}(B, [f_1, \dots, f_i, \dots, f_n]) = [f_i, \dots, f_n]$.
- (6) The expand operator $\text{expand}_{\mathbb{C}}$ restores the removed part of the call stack: $\text{expand}_{\mathbb{C}}([f_1, \dots, f_i, \dots, f_n], B, [f_i, \dots, f_s]) = [f_1, \dots, f_i, \dots, f_s]$.
- (7) The *call-stack analysis* does not track variables, but the procedure scopes themselves. Thus the rebuild operator is defined as: $\text{rebuild}_{\mathbb{C}}(e_{call}, e_I, e_O) = e_O$.

Value-CPA. The CPA with BAM operators for value analysis $\mathbb{B} = (D_{\mathbb{B}}, \sim_{\mathbb{B}}, \text{merge}_{\mathbb{B}}, \text{stop}_{\mathbb{B}}, \text{reduce}_{\mathbb{B}}, \text{expand}_{\mathbb{B}}, \text{rebuild}_{\mathbb{B}})$ explicitly tracks the assignments of variables. The CPA is used as described in previous work [12, 17] and extended by BAM operators.

- (1) The domain $D_{\mathbb{B}} = (C, \mathcal{E}_{\mathbb{B}}, \llbracket \cdot \rrbracket)$ is based on the semi-lattice $\mathcal{E}_{\mathbb{B}} = (V, \sqsubseteq_{\mathbb{B}})$ for the set $V = (X \rightarrow \mathbb{Z})$ of partial functions that model abstract variable assignments for a set X of variables and the set \mathbb{Z} of integer values. We use $v(x)$ to denote the value of a variable $x \in X$ for an abstract variable assignment $v \in V$, and we use $\text{dom}(v)$ to denote the set of variables for which v assigns a value, that is, $\text{dom}(v) = \{x \mid (x, \cdot) \in v\}$. The partial order $\sqsubseteq_{\mathbb{B}} \subseteq V \times V$ is defined as: $v \sqsubseteq v'$ if $\text{dom}(v') \subseteq \text{dom}(v)$ and $v(x) = v'(x)$ is

satisfied for all $x \in \text{dom}(v')$. The top element $\top_{\mathbb{E}} \in V$ (least upper bound) denotes the abstract variable assignment with no specific value for any variable: $\top_{\mathbb{E}} = \{\}$. The join operator $\sqcup_{\mathbb{E}} : E \times E \rightarrow E$ is based on the partial order and returns the least upper bound of its operands. The concretization function $\llbracket \cdot \rrbracket : V \rightarrow 2^C$ returns the meaning for an abstract variable assignment.

- (2) The transfer relation $\rightsquigarrow_{\mathbb{E}}$ has the transfer $v \xrightarrow{g}_{\mathbb{E}} v'$ for a CFA edge $g = (\cdot, \text{op}, \cdot)$ and two abstract variable assignments v and v' , if one of the following conditions is satisfied (given a predicate p and an abstract variable assignment v , we define $\phi(p, v) := p \wedge \bigwedge_{x \in \text{dom}(v)} x = v(x)$):
 - (a) $\text{op} = \text{assume}(p)$ and predicate $\phi(p, v)$ is satisfiable, and v' is defined as follows: $(x, c) \in v'$ if c is the only satisfying assignment for variable x of the predicate $\phi(p, v)$, or
 - (b) $\text{op} = (w := \text{exp})$ and $(x, c) \in v'$ if either $(x \neq w$ and $(x, c) \in v)$ or $(x = w$ and c is the only satisfying assignment for variable x' of the predicate $\phi(x' = \text{exp}, v)$).
- (3) The merge operator $\text{merge}_{\mathbb{E}} = \text{merge}^{sep}$ does not combine abstract states.
- (4) The termination check $\text{stop}_{\mathbb{E}} = \text{stop}^{sep}$ returns whether a covering abstract state was already reached before.
- (5) The reduce operator $\text{reduce}_{\mathbb{E}}$ only keeps abstract assignments of variables that are accessed in the block's context: $\text{reduce}_{\mathbb{E}}(B, e_I) = \{(x, c) \in e_I \mid x \text{ used in } B\}$.
- (6) The expand operator $\text{expand}_{\mathbb{E}}$ restores the assignments that were removed by $\text{reduce}_{\mathbb{E}}$ from the initial abstract state: $\text{expand}_{\mathbb{E}}(e_I, B, e_O) = \{(x, c) \in e_I \mid x \text{ not used in } B\} \cup e_O$.
- (7) For the rebuild operator $\text{rebuild}_{\mathbb{E}}$, we define *global* variables as variables declared in the global scope and the rest as *local* variables, i. e., variables declared in a local procedure scope. After leaving a (recursive) procedure call, the operator $\text{rebuild}_{\mathbb{E}}$ considers local variables from the calling scope, and global variables and the return variable¹ from the exited procedure scope: $\text{rebuild}_{\mathbb{E}}(e_{\text{call}}, e_I, e_O) = \{(x, c) \in e_{\text{call}} \mid \neg \text{isGlobal}(x) \wedge \neg \text{isReturn}(x)\} \cup \{(x, c) \in e_O \mid \text{isGlobal}(x) \vee \text{isReturn}(x)\}$.

Because global variables can be assigned during the procedure's execution, they are not reset to their assigned value from before the procedure's execution; their values are taken from the abstract state e_O at the procedure's exit location.

Note that with these definitions of $\text{reduce}_{\mathbb{E}}$ and $\text{expand}_{\mathbb{E}}$, the *value analysis* of a procedure block is not completely detached from the calling context, because a block abstraction for this domain depends on the input values of variables accessed in the block. For procedure blocks, a block abstraction for a function call can be taken from the BAM cache whenever the function arguments and global variables have identical values.

Predicate-CPA. The CPA with BAM operators for predicate analysis $\mathbb{P} = (D_{\mathbb{P}}, \rightsquigarrow_{\mathbb{P}}, \text{merge}_{\mathbb{P}}, \text{stop}_{\mathbb{P}}, \text{reduce}_{\mathbb{P}}, \text{expand}_{\mathbb{P}}, \text{rebuild}_{\mathbb{P}})$ uses predicates to track variables and their values [8, 57]. For this analysis a set \mathcal{P} of predicates is used, which can be incrementally computed in a CEGAR loop [27] that is applied on top of the CPA

algorithm. In this description, we do not go into detail on how to determine useful predicates, but assume that the predicates are already available, e. g., by applying an existing refinement strategy [10, 16]. The refinement procedure of the *predicate analysis* computes interpolants that match the structure of the procedure blocks [38] and allow an interprocedural analysis.

For each block B , we partition the set \mathcal{P} of predicates into two disjoint sets $\mathcal{P}_B = \{p \in \mathcal{P} \mid p \text{ relevant for } B\}$ and $\mathcal{P}_{-B} = \mathcal{P} \setminus \mathcal{P}_B$. A predicate $p \in \mathcal{P}$ is *relevant for* B if it contain variables that are accessed in the block. The partition \mathcal{P}_{-B} contains the rest of \mathcal{P} .

- (1) The domain $D_{\mathbb{P}} = (C, \mathcal{E}_{\mathbb{P}}, \llbracket \cdot \rrbracket)$ is based on the set C of concrete states, the lattice $\mathcal{E}_{\mathbb{P}} = (E, \sqsubseteq_{\mathbb{P}})$, and a concretization function $\llbracket \cdot \rrbracket : E \rightarrow C$. The lattice consists of abstract states $e \in E$ that are tuples $(\psi, l^{\psi}, \varphi) \in (\mathcal{P} \times (L \cup \{l_{\top}\}) \times \mathcal{P})$. The *abstraction formula* ψ is a boolean combination of predicates from \mathcal{P} and has been computed at the program location l^{ψ} . The *path formula* φ represents (the disjunction of) all paths from l^{ψ} to the abstract state e . The partial order $\sqsubseteq \subseteq E \times E$ is defined for any two abstract states $e_1 = (\psi_1, l^{\psi_1}, \varphi_1)$ and $e_2 = (\psi_2, l^{\psi_2}, \varphi_2)$ as: $e_1 \sqsubseteq e_2$ if $(e_2 = \top_{\mathbb{P}}) \vee ((l^{\psi_1} = l^{\psi_2}) \wedge (\psi_1 \wedge \varphi_1 \Rightarrow \psi_2 \wedge \varphi_2))$. The top element is $\top_{\mathbb{P}} = (\text{true}, l_{\top}, \text{true})$. The join operator $\sqcup : E \times E \rightarrow E$ is based on the partial order and returns the least upper bound of its operands.
- (2) The transfer relation $\rightsquigarrow_{\mathbb{P}}$ has the transfer $e \xrightarrow{g}_{\mathbb{P}} e'$ for an edge $g = (\cdot, \text{op}, l')$ and two abstract states $e = (\psi, l^{\psi}, \varphi)$ and $e' = (\psi', l^{\psi'}, \varphi')$, if

$$(\psi', l^{\psi'}, \varphi') = \begin{cases} (\text{true}, l', (SP_{\text{op}}(\varphi) \wedge \psi)^{\Pi}) & \text{if } \text{blk}(e, g) \\ (\psi, l', SP_{\text{op}}(\varphi)) & \text{otherwise} \end{cases}$$
 where $SP_{\text{op}}(\varphi)$ denotes the strongest post-condition of a given path formula φ for an operation op . The choice of computing a boolean predicate abstraction depends on the configurable operator blk . For our work it returns *true* at least for procedure calls, procedure entries, and procedure exits. The boolean predicate abstraction $(\cdot)^{\Pi}$ computes the strongest boolean combination of predicates from \mathcal{P} .
- (3) The merge operator $\text{merge}_{\mathbb{P}} : E \times E \rightarrow E$ combines the two abstract states $e_1 = (\psi_1, l^{\psi_1}, \varphi_1)$ and $e_2 = (\psi_2, l^{\psi_2}, \varphi_2)$ according to their last abstraction computation: $\text{merge}(e_1, e_2) = \begin{cases} (\psi_2, l^{\psi_2}, \varphi_1 \vee \varphi_2) & \text{if } (\psi_1 = \psi_2) \wedge (l^{\psi_1} = l^{\psi_2}) \\ e_2 & \text{otherwise} \end{cases}$
- (4) The termination check $\text{stop}_{\mathbb{P}} = \text{stop}_{sep}$ returns whether a covering abstract state was already reached before.
- (5) For an abstract state $e_I = (\psi_I, l^{\psi_I}, \text{true})$ at a block entry, the operator $\text{reduce}_{\mathbb{P}}$ computes the set $\mathcal{P}_{-B} := \{p_1, \dots, p_i\}$ of predicates that are irrelevant for the block abstraction and removes them from the abstraction formula: $\text{reduce}_{\mathbb{P}}(B, e_I) = ((\exists p_1, \dots, p_i : \psi_I), l^{\psi_I}, \text{true})$.²
- (6) The operator $\text{expand}_{\mathbb{P}}$ reverts the operation $\text{reduce}_{\mathbb{P}}$, it computes the set $\mathcal{P}_B := \{p_{i+1}, \dots, p_n\}$ of predicates, and restores the full set of predicates $\mathcal{P} = \mathcal{P}_{-B} \cup \mathcal{P}_B$ for an output state $e_O = (\psi_O, l^{\psi_O}, \text{true})$ as follows: The abstraction formula ψ_O

¹Our implementation introduces an additional variable to capture the return value, such that we are able to reference it here as well.

²We represent the abstraction formula ψ in a way that makes it easy to remove elements from \mathcal{P} in an atomic way from an abstraction formula. (We represent ψ as a binary decision diagram (BDD) whose boolean variables represent predicates from \mathcal{P} .)

is extended by the remaining part of the initial abstraction formula ψ_I :

$$\text{expand}_{\mathbb{P}}(e_I, B, e_O) = ((\exists p_{i+1}, \dots, p_n : \psi_I) \wedge \psi_O, l^{\psi_O}, \text{true}).$$

- (7) The operator $\text{rebuild}_{\mathbb{P}}$ is based on the procedure-call state $e_{\text{call}} = (\psi_{\text{call}}, l^{\psi_{\text{call}}}, \text{true})$, the (not reduced) procedure-entry state $e_I = (\psi_I, l^{\psi_I}, \text{true})$, and the expanded procedure-exit state $e_O = (\psi_O, l^{\psi_O}, \text{true})$. The path formula φ_{call} represents the CFA edge that is the procedure entry edge between the program locations of the abstract states e_{call} and e_I and represents the encoding of all assignments of the actual arguments to the formal parameter variables. The operator $\text{rebuild}_{\mathbb{P}}$ computes the predicate abstraction for the conjunction of the abstractions before and after the procedure call and the parameter assignment:

$$\text{rebuild}_{\mathbb{P}}(B, e_{\text{call}}, e_I, e_O) = (\psi_{\text{call}} \wedge \varphi_{\text{call}} \wedge \psi_O)^{\Pi}.$$

Interval-CPA. The CPA with BAM operators for interval analysis $\mathbb{I} = (D_{\mathbb{I}}, \rightsquigarrow_{\mathbb{I}}, \text{merge}_{\mathbb{I}}, \text{stop}_{\mathbb{I}}, \text{reduce}_{\mathbb{I}}, \text{expand}_{\mathbb{I}}, \text{rebuild}_{\mathbb{I}})$ tracks variables and the range (interval) of their possible assigned values. The interval analysis is similar to the value analysis, which can be seen as a special case using intervals containing only one value. The coverage relation between intervals is based on the inclusion of intervals (instead of equality of values). We omit the detailed definition here to keep the reader focused on our approach.

4.1 Soundness of Reduce and Expand Operator for the Given Domains

For each of the described domains, the soundness criterion of the whole interprocedural analysis is based on the soundness of the CPA algorithm itself (which we assume as basis) as well as on the properties of the specific operators reduce and expand. For a sound analysis, the abstract states that would have been reached without applying a block abstraction (i. e., only applying the wrapped CPA \mathbb{W}) need to be a subset of the states reached with an application of the corresponding block abstraction, i. e., using block abstractions can only be less precise than a wrapped analysis, but never cut off a reachable part of the abstract state space.

The transfer relation $\rightsquigarrow_{\text{BAM}}$ for an abstract state $e \in E$ satisfies the relation $\{e' \in E \mid e \rightsquigarrow_{\mathbb{W}} e'\} \subseteq \{e'' \in E \mid e \rightsquigarrow_{\text{BAM}} e''\}$. Based on the definition of $\rightsquigarrow_{\text{BAM}}$ (Sect. 3.2), the interesting case appears when applying a block abstraction. Thus, the concrete implementation of the operators reduce and expand must satisfy the following condition for all blocks B : $\{e' \in E \mid e \rightsquigarrow_{\mathbb{W}} e'\} \subseteq \{\text{expand}(e, B, e_O) \in E \mid \text{reduce}(B, e) \rightsquigarrow_{\mathbb{W}} e_O\}$.

For the call-stack analysis, each abstract call-stack state after an application of a block abstraction exactly matches the call-stack state without such a block abstraction. To prove this, just extend each call stack during the block analysis with the removed part $[f_1, \dots, f_{i-1}]$ from the reduce operation. For the value analysis (and based on a programming language without pointer handling), the same proof can be applied: Removing assignments from abstract states and restoring them later results in an abstract state that matches the state when not applying a block abstraction computation. A detailed soundness proof for the predicate domain is given in the literature [57]. Removing irrelevant predicates $P_{\rightarrow B}$ and conjuncting those predicates when applying the block abstraction does

only make the analysis more imprecise, but does not reduce the reachable abstract state space.

4.2 Embedding BAM Interprocedural in CEGAR

The framework CPACHECKER defines BAM as a CPA and allows to combine the CPA algorithm with other algorithms, like CEGAR [27], which allows to refine the granularity of the abstract analysis based on information extracted from infeasible program paths. Additional operators for the refinement step in CEGAR are also defined in a domain-independent manner and available in the framework. In our case, the CEGAR algorithm can wrap the CPA algorithm and the analysis of BAM can benefit from this. Whenever BAM finds a property violation, the reachability analysis and the fixed-point algorithm terminates and the surrounding CEGAR algorithm checks the error path for feasibility. If necessary, CEGAR refines the precision, and BAM with the fixed-point algorithm is re-started with the updated precision.

In case of the predicate analysis, the refinement procedure computes tree interpolants [20, 38] according to procedure scopes, i. e., for each entered (and exited) procedure scope along an infeasible error path, a new subtree for the tree interpolation problem is constructed. For other analyses, like value analysis, the refinement of recursive procedures does not need special handling. In this case, a refinement strategy for sequential error paths [17] is sufficient.

4.3 Detailed Description of the Example

In the following, we provide deeper insights for the previously given example program (see Sect. 1) in Fig. 1, to show the control flow of BAM with the fixed-point algorithm when using the predicate analysis. We combine the previously defined Callstack-CPA \mathbb{C} and the Predicate-CPA \mathbb{P} , i. e., the transfer relation, coverage check, reduce, expand, and rebuild operators are applied in both domains.

Figure 5 shows the abstract states that are reached in the first two iterations of the fixed-point algorithm, which terminates after the second iteration. The labeling of each abstract state consists of the program location (circled number in first line), the call stack (second line), and the abstraction formula of the predicate analysis (third line). To keep the figure readable, we dismiss the call stack and abstraction formula whenever there is no change in the abstract state. Outside the upper left corner of each node, we annotate e_i , where index i refers to the exploration strategy and control flow of the analysis.

The operators reduce, expand, and rebuild show their effect at the program locations 11 and 16, which are the input and output locations of the procedure block B_{sum} . For example, the operator $\text{reduce}_{\mathbb{C}}$ of the call-stack analysis removes of all procedure scopes except the most local one from the call stack. The operator $\text{expand}_{\mathbb{C}}$ restores the whole call stack when the analysis leaves the block. The effect of the $\text{rebuild}_{\mathbb{P}}$ at program location 16 will be described below.

Initialization. We assume that the initial cache and the stack of BAM are empty and the following set of predicates is defined as precision: $\mathcal{P} := \{\text{ret} = m_p + n_p, \text{ret} = a + b, m = m_p \wedge n = n_p\}$. The predicate analysis uses the symbols m_p , n_p , and ret to encode parameter assignments at function entry and the return value. Such predicates can be generated via an interpolation procedure from previously found infeasible error paths in the context of CEGAR.

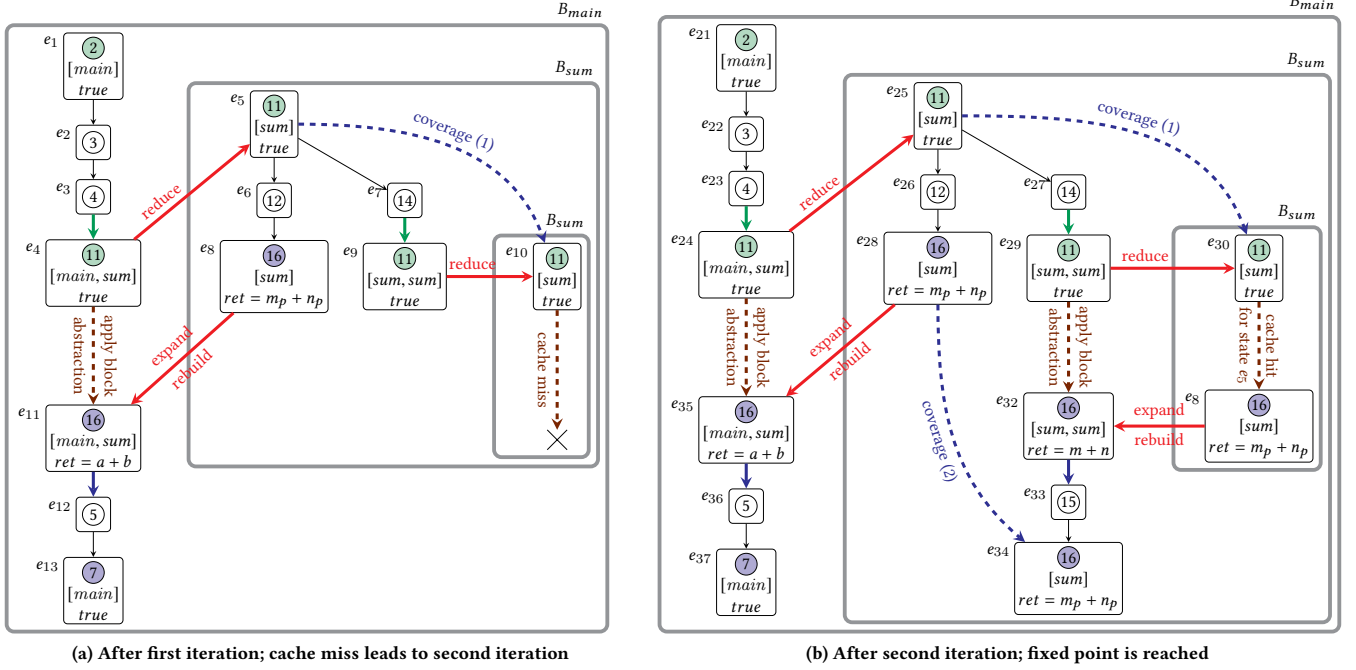


Figure 5: Graph of reached abstract states after the first two fixed-point iterations

For simple programs (like this example) they match the expected procedure summary. In general, the analysis might need several iterations of CEGAR to obtain a sufficient precision. In this example, we concentrate on the rebuild operator. All predicates are relevant for the block B_{sum} , i. e., $\mathcal{P}_B = \mathcal{P}$, i. e., the reduce and expand operators for *predicate analysis* will keep the abstraction formula unchanged.

First Iteration. The result of the first iteration of the fixed-point loop is shown in Fig. 5a. The analysis starts with the initial abstract state e_1 at program location 2, entering the main block B_{main} and pushing e_1 (as e_i in Alg. 2) onto the BAM stack. The recursive procedure block B_{sum} is analyzed for the first time at the procedure call from program location 4 to program location 11, where BAM starts a new sub-analysis with state e_4 (as e_I in Alg. 2) for the block B_{sum} . The reduction removes the suffix *main* of the call stack and keeps the abstraction formula *true*. The abstract state e_5 (as e_i in Alg. 2) is pushed onto the BAM stack. When the procedure block B_{sum} is entered the second time (procedure call at program location 14 for state e_9), the reduced abstract state e_{10} is compared with elements in the BAM stack. The coverage relation (Alg. 2, line 2) is satisfied. BAM has no computed procedure summary in the cache and returns an empty set of reachable abstract states (line 6 of Alg. 2). The flag *fixedpointReached* is set to *false* in line 7 of Alg. 2. The analysis continues with the exploration of the non-recursive branch of the procedure. When leaving block B_{sum} , the block's summary is inserted into the cache, i. e., the block abstraction from the abstract state e_5 towards the abstract state e_8 (as e_O in Alg. 2) is stored for later usage in the BAM cache. For the predicate analysis, the summary of the block is the abstraction formula $ret = m_p + n_p$, which describes the equality of the sum of the two formal function parameters with the return value.

The rebuild operator $rebuild(B, e_3, e_4, e_8)$ restores information from the calling context. Using the abstraction formula $\psi_3 := true$, the parameter assignment from the procedure call $\varphi_{call} := (a = n_p \wedge b = m_p)$, and the block summary $\psi_8 := (ret = m_p + n_p)$, the rebuild operator $rebuild_{\mathbb{P}}$ computes $(\psi_3 \wedge \varphi_{call} \wedge \psi_8)^{\Pi} = (ret = a + b)$. That is, based on the given predicates for e_{11} , $\Pi(e_{11}) = \{ret = a + b\}$, the procedure is summarized by $ret = a + b$, which describes the equality of the sum of the two actual function arguments with the return value. We do not describe internals of *predicate abstraction* here, but refer to the literature [16]. No property violation is found along the path until state e_{13} , i. e., the branching towards program location 6 is not satisfiable, and the fixed-point computation continues.

Second Iteration. The initial steps of the second iteration are similar to the first iteration. After a few steps, the stack consists of the abstract states e_{21} and e_{25} . A different control flow appears when the analysis reaches the recursive procedure call again at state e_{30} , with a coverage relation for the abstract state e_{25} because it is part of the BAM stack. Now we get a cache hit for the previously computed block abstraction between state e_5 and state e_8 and apply the procedure summary to skip the recursive procedure call (line 4 of Alg. 2). Using the abstraction formula $\psi_{27} := true$, the parameter assignment from the procedure call $\varphi_{call} := (n = n_p \wedge m = m_p)$, and the block summary $\phi_8 := (ret = m_p + n_p)$, the rebuild operator $rebuild_{\mathbb{P}}$ computes $(\psi_{27} \wedge \varphi_{call} \wedge \phi_8)^{\Pi} = (ret = m + n)$. When leaving the procedure block, our approach (Alg. 2, line 19) checks for new (not yet covered) abstract states. In this example, state e_{34} is already covered by state e_{28} , thus the fixed-point algorithm terminates after this iteration. As the property violation at program location 6 is not reachable, the program is verified.

5 EXPERIMENTAL EVALUATION

We evaluate BAM Interprocedural for several domains and show that it is competitive with existing approaches. We divide the evaluation according to three claims. For both claims, we use a benchmark set of non-recursive and recursive programs and provide the effectivity (number of solved problems) and performance (runtime) of our implementation, using several analyses of CPACHECKER and other verification tools.

Claim I: Domain-Independence and Modularity. We claim that our interprocedural approach is domain-independent and can be implemented in a modular way as described in Sect. 3, such that the development and integration overhead for an existing analysis in the framework CPACHECKER is quite small. To evaluate the claim, we apply the approach to several abstract domains, show that the analysis works, and compare different analyses of CPACHECKER against each other.

Claim II: Effectiveness and Efficiency (Part 1). We claim that our approach—despite the modular design—does not cause large performance overheads in an analysis. To evaluate the claim, we compare benchmark results against several state-of-the-art verification tools that are able to verify programs with recursive procedures.

Claim III: Effectiveness and Efficiency (Part 2). We claim that our approach is comparable to intraprocedural analyses within the same framework. To evaluate the claim, we apply different analyses to a larger set of recursive and non-recursive benchmark tasks and compare benchmark results from our interprocedural approach against intraprocedural analyses with and without BAM.

5.1 Benchmark Programs and Setup

We use verification tasks from the SV-COMP '20 [5] benchmark set³, including tasks with and without recursive function calls from categories *ReachSafety-Bitvectors*, *ReachSafety-ControlFlow*, *ReachSafety-Loops*, *ReachSafety-ProductLines*, and *ReachSafety-Recursive*. Most recursive programs are generic and allow to easily scale the programs to deeper recursion; they include recursive algorithms, e. g., *Fibonacci*, *Ackermann*, *Towers of Hanoi*, and *McCarthy91*. The non-recursive programs use integer arithmetics and avoid heap-related data-structures.

All experiments were performed on machines with a 3.4 GHz Quad Core CPU and 33 GB of RAM. The operating system was Ubuntu 20.04 (64 bit) with Linux 5.4.0. A CPU time limit of 15 min and a memory limit of 15 GB were used, which is the established standard from SV-COMP. Measurements and resource limits were managed by BENCHEXEC [18].

5.2 Results and Discussion

Claim I. We implemented our domain-independent approach in CPACHECKER for several domains, including *value analysis*, *predicate analysis*, and *interval analysis*. In addition, we evaluated a reduced product [14, 32] of value and predicate analysis. We used CPACHECKER in version 1.9, which also participated in SV-COMP '20. CPACHECKER

Table 1: Results for the comparison of BAM Interprocedural combined with different abstract domains in CPACHECKER on category *ReachSafety-Recursive* of SV-COMP

Domain	CPU time (s)	Proofs	Bugs
Value	924	31	37
Predicate	3 440	29	37
Interval	849	36	38
Value + Predicate	1 690	37	43

Table 2: Results for the comparison of different verifiers on category *ReachSafety-Recursive* of SV-COMP

Verifier	CPU time (s)	Proofs	Bugs
CBMC	662	32	47
CPACHECKER (SV-COMP '20)	2 180	37	46
DIVINE	1 190	32	42
ESBMC	941	33	47
MAP2CHECK	23 600	34	37
PeSCo	3 130	37	46
PINAKA	237	31	31
SYMBIOTIC	138	33	45
UUTOMIZER	2 160	41	37
UKOJAK	1 010	19	28
UTAIPAN	6 210	42	37
VERIABS	7 630	41	46
VERIFUZZ	1 960	0	45

was chosen as the implementation platform because it has a configurable and modular design that is easy to extend by new concepts, has a considerable user base, and is well maintained.⁴

Table 1 compares BAM Interprocedural for four domains (one of them being a product), by providing the CPU time (in seconds, with three significant digits) needed by the verifiers for all correctly solved verification tasks and the number of correctly solved tasks, divided into proofs and bugs found in the category *ReachSafety-Recursive* of SV-COMP.

Claim II. We provide the results of state-of-the-art software verifiers, which participated in SV-COMP '20 [5]⁵. We compare 13 verifiers that participated successfully in the category *ReachSafety-Recursive* of SV-COMP. This includes the predicate-based verifiers CPACHECKER [35, 56], PeSCo [34, 49] and ULTIMATE AUTOMIZER [37, 39], the bounded model checkers CBMC [29, 42] and ESBMC [36, 45], the symbolic-execution tool SYMBIOTIC [24, 25], as well as the SMT-based tool MAP2CHECK [51, 52]. The binary archives of all verifiers are publicly available.⁶ The data are extracted from the published SV-COMP '20 results [6].

Table 2 provides the sum of CPU time needed by the verifiers for all correctly solved verification tasks, and the number of correctly solved tasks, divided into proofs and bugs found. The configuration used by verifier CPACHECKER (SV-COMP '20) combines *value analysis* and *predicate analysis* within our interprocedural approach (same configuration as in the last entry of Table 1), which is automatically selected as the strategy to verify recursive programs [7]. The performance of the tool with our approach (CPACHECKER) also shows that

⁴<https://www.openhub.net/p/cpachecker>

⁵<https://sv-comp.sosy-lab.org/2020/systems.php>

⁶<https://gitlab.com/sosy-lab/sv-comp/archives-2020/-/tree/svcomp20>

³<https://github.com/sosy-lab/sv-benchmarks/tree/svcomp20>

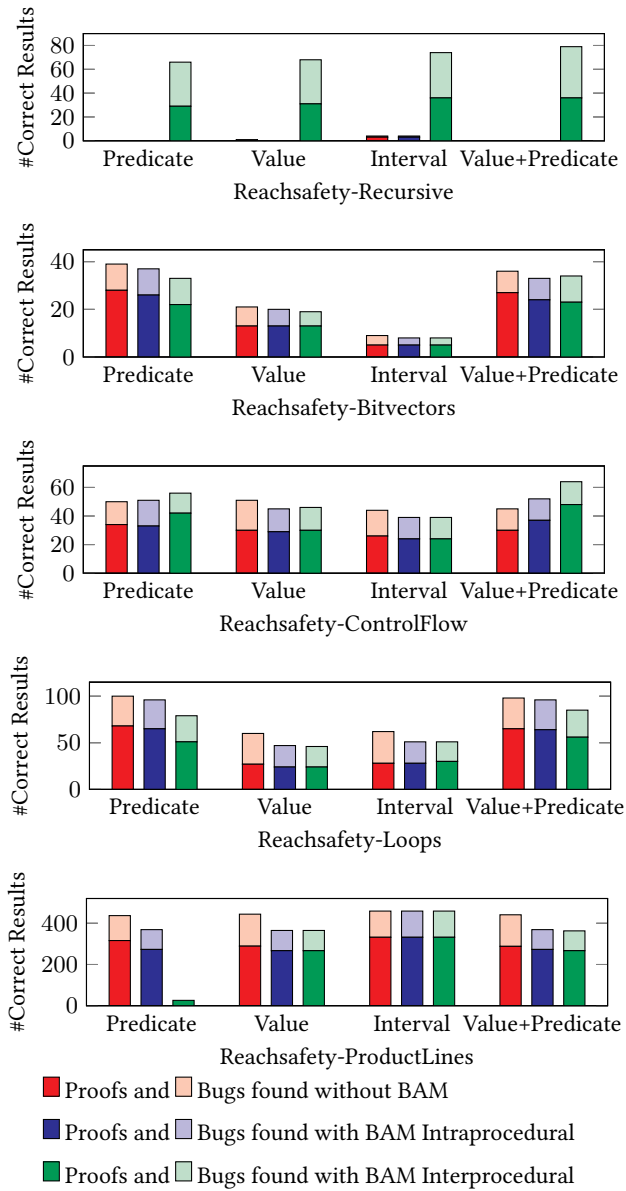


Figure 6: Results for different benchmark categories for the comparison of different abstract domains without BAM, with BAM Intraprocedural, and with BAM Interprocedural in CPAchecker

although modular and domain-independent, it is competitive with completely different tools and approaches in terms of *effectiveness* and *efficiency*: BAM Interprocedural solves about as many tasks as the other tools within reasonable CPU time. None of the tools managed to verify all tasks, and there are several tasks in the given benchmark set that could not be solved by any verifier.

Claim III. As CPAchecker is the configurable program analysis framework, different domain-independent intraprocedural analyses based on the CPA concept are available, such as the default analysis without BAM and its combination with BAM. Figure 6

compares those algorithms with our new approach of BAM Interprocedural. Each analysis is combined with four different domains (one of them being a product). We provide the number of correctly solved tasks, divided into proofs and bugs found. Each category of SV-COMP '20 is given separately, such that the strengths of the algorithms are visible. In contrast to the existing intraprocedural approaches without and with BAM, the new approach supports the interprocedural analysis of recursive procedures for all three domains separately as well as for a combination of domains and leads to good results in the category *ReachSafety-Recursive*. For all other categories, the results are comparable over all approaches. Only for the predicate domain, the result for the tasks in category *ReachSafety-ProductLines* is worse. The reason for the result in this single category is caused by a valid, but unfitting refinement step (i. e., a suboptimal heuristic in the SMT solver), that causes expensive unrolling of the program. As many tasks in this category are similar, most results are affected. For *value analysis*, *interval analysis*, and also for the analysis based on value and predicate domain together, the new approach performs approximately as good as the existing approaches without or with BAM.

6 CONCLUSION

We have presented BAM Interprocedural, a novel approach to interprocedural program analysis. The new approach is **modular** and **domain-independent**, because it is not integrated in a specific program analysis but wraps an existing analysis. In other words, given an arbitrary abstract domain for *intra*-procedural data-flow analysis, we can turn it into an *inter*-procedural analysis without much (a) development work and (b) performance overhead. We have illustrated in detail how to make *predicate analysis* and *value analysis* interprocedural. Our implementation and experiments show that BAM Interprocedural works well for four different program analyses. The new approach supports **recursive** procedures, because it is not bounded to a fixed number of procedure scopes. We showed the effectiveness on the benchmark set of recursive programs from SV-COMP '20: the approach is able to successfully verify recursive procedures. The new approach is **efficient**, because it is integrated into BAM and does not add much overhead on top of the wrapped abstract domain. Compared to other software verifiers, the new implementation is competitive. Due to the modular approach, the effectiveness and efficiency heavily depends on the wrapped program analysis. Our results are promising and there is potential for optimization in our implementation. We plan to specify the operator rebuild for further domains like binary decision diagrams, symbolic memory graphs, or octagons, e. g., to analyze more difficult memory-accesses in recursive programs.

We hope that other researchers and developers of verification tools can benefit from our approach because it separates the concern of making an analysis interprocedural from the actual work on implementing and improving abstract domains.

Data Availability Statement. All benchmark tasks for evaluation, configuration files, a ready-to-run version of our implementation, and tables with detailed results are available in our reproduction package [11]. The source code of our extensions to the open-source verification framework CPAchecker [15] is available in the project repository; see <https://cpachecker.sosy-lab.org>.

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